Stochastic Optimization - 2 hours

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This exam has two independent parts. Course documents authorized. No communication with other people.

1 Energy production (8 points)

Consider an energy producer which has a random demand d to meet or excess. He can produce energy q (in MWh) for a cost q^2 through a thermal engine that has to be set in advance. If the production is not enough he can buy energy b, on the spot (i.e. even at the last minute) the missing energy for $p_1 b$, with $0 \leq b \leq 2.5MWh$.

Demand is assumed to be $1MWh$ with probability 0.5, $2MWh$ with probability 0.25 and $3MWh$ with probability 0.25.

1. (1 point) Justify that this problem is a two-stage stochastic program by specifying first and second stage variable and write the extensive formulation of this problem.
2. (1.5 points) Write a time-decomposition of the problem that is in a relatively complete recourse setting.
3. (1.5 points) Assuming that $p_1 = 4$, compute the anticipative lower bound.
4. (2 points) Can we apply the Progressive Hedging algorithm to this problem ? If so give the master problem and slave problems at iteration k. If not Justify.
5. (2 points) Slightly adapt the L-Shaped method to this problem. Gives the master problem and slave problems (in primal form) at iteration k (multicut version).

2 A stochastic SOCP (13 points)

A second order cone program (SOCP) is an optimization problem of the form

$$
\begin{array}{llll}
(SOCP) & \min_{z \in \mathbb{R}^n} & q^\top z & \\
& s.t. & \|A_i z + b_i\|_2 \leq c_i^\top z + d_i & \forall i = 1, .., k \\
& & A_0 z = b_0 & \\
& & A_1 z \leq b_1 &
\end{array}
$$

In particular every linear program are SOCP, and you can add some constraints with $\|\|_2$ norm.

We now consider the following stochastic optimization problem (all equalities hold almost surely)

$$
\begin{array}{llll}
(P) & \min & c^\top x_0 + \mathbb{E}\left[\sum_{t=0}^{T-1} q_t \|\boldsymbol{u}_t\|_2\right] & \\
& s.t. & \boldsymbol{x}_{t+1} = \boldsymbol{x}_t + H\boldsymbol{u}_t & \forall t = 0, ..., T-1 \\
& & \|\boldsymbol{x}_T\| \leq R & \mathbb{P} - a.s. \\
& & A\boldsymbol{x}_t + B\boldsymbol{u}_t \leq \boldsymbol{b}_t & \\
& & \sigma(\boldsymbol{x}_t, \boldsymbol{u}_t) \subset \sigma(\boldsymbol{b}_0, \ldots, \boldsymbol{b}_t) &
\end{array}
$$

Where $\boldsymbol{x}_t \in \mathbb{R}^3$ and $\boldsymbol{u}_t \in \mathbb{R}^{10}$. We assume that the noise $(\boldsymbol{b}_t)_t$ is a sequence of independent random variables, such that, for each t, $\boldsymbol{b}_t$ has a finite support of size 4.

1. We start with some generic comments on (P).
   (a) (0.5 points) Show that an SOCP is a convex problem
   (b) (0.5 points) What is the state, the control, the noise affecting (P) ?
2. In this question we assume that $T = 2$.
   (a) (3 points) Write the extensive formulation of problem (P) as a finite dimensional SOCP. Give the number of optimization variables and the number of conic constraints with their size (i.e the dimension of the vector inside the norm).
   (b) (2 points) Write the two-stage approximation $(2SA)$ of (P) as a SOCP. Is it simpler to solve than the full problem ?
   (c) (1 point) Would solving a SAA version of $(2SA)$ be faster ? Justify.
3. We now consider the case of large T.
   (a) (2 points) Give a Bellman operator $\mathcal{B}$ associated to problem (P), and the dynamic programming equation. Do not forget the initialization of the recursion with the final cost. (You might write a special problem for $t = 0$).
   (b) (0.5 points) Is computing $\mathcal{B}_t(V_{t+1})(x_t)$ equivalent to solving a SOCP ?
   (c) (2.5 points) Assuming that you have an efficient SOCP solver, give a pseudo-code for solving (P) by Dynamic Programming through a discretization / interpolation approach. Is it reasonable ?
   (d) (1 point) Can you use SDDP to solve (P) ? Justify.