

Homework — Production Planning

Introduction to Optimization

Autumn 2026

1 Objective and practical information

The objective of this homework is to build, solve, and analyse a small production-planning model, first as a linear program (LP) and then as a mixed-integer linear program (MILP).

The submission may be prepared in pairs, but the short oral assessment is individual. Documentation, coding assistants, and large language models may be used while preparing the homework, but not during the assessment; in either case you remain responsible for understanding the mathematical model and the code you submit.

A Julia notebook using JuMP with the open-source solver HiGHS is recommended. Other modeling languages and solvers are allowed.

For the MILP part, the relevant material is the chapter on mixed-integer linear programming in the course notes. The homework itself requires only elementary binary-variable constructions.

2 Production data

A company manufactures six products, denoted $P_1, \dots, P_6$. Production is planned on a weekly basis. Each unit produced generates a contribution margin and consumes three limited resources: machining time, assembly time, and finishing time.

The unit data are:

Product	Unit margin (€)	Maximum sales	Hours per unit		
			Machining	Assembly	Finishing
P_1	32	80	2	4	1
P_2	40	70	3	2	2
P_3	38	90	2	3	2
P_4	55	60	4	2	3
P_5	53	75	3	4	1
P_6	60	50	5	3	2

The available resources for the coming week are:

Resource	Available hours
Machining	520
Assembly	480
Finishing	300

Production quantities may be treated as continuous.

3 Linear production model

Initially assume that producing a product carries no fixed cost and that products can be selected independently: any quantity between zero and the maximum sales quantity may be produced.

Q1. Define appropriate decision variables.

Q2. Formulate a linear optimization problem that maximizes the weekly contribution margin.

Q3. Implement and solve it. Report the optimal objective value, the quantity produced of each product, and, for each resource, its total use, available quantity, slack, and whether the resource constraint is active.

Present the resource information as a table of the form

Resource	Used (h)	Available (h)	Slack (h)	Active?
Machining	...	...	...	...
Assembly	...	...	...	...
Finishing	...	...	...	...

4 Operational production rules

In reality, producing a product requires a setup operation. A fixed setup cost is incurred whenever the product is produced, independently of the quantity produced. Moreover, once production of a product is launched, the production run must reach a minimum size.

Product	Fixed setup cost (€)	Minimum production if activated
P_1	400	20
P_2	450	20
P_3	500	25
P_4	700	15
P_5	500	20
P_6	600	10

The weekly profit is now the contribution margin generated by production minus the setup costs of the products that are activated. There is also one technological dependency:

Product P_4 may be produced only during a week in which product P_1 is also produced.

Q4. Introduce the additional decision variables that are needed.

Q5. Formulate a MILP incorporating the fixed setup costs, the minimum production quantities, and the technological dependency between P_4 and P_1 .

5 Numerical analysis

The purpose of this part is not to perform an extensive computational study, but to use the optimization model to understand the production decision.

5.1 Required analysis

Solve the following four models successively:

M1. the LP of Question 3;

M2. that LP with product-activation variables and fixed setup costs, but without minimum-production requirements and without the technological dependency;

M3. M2 with the minimum-production requirements;

M4. M3 with the P_4/P_1 dependency, that is, the complete MILP of Question 5.

For each model, report the optimal objective value, the production quantities, the products that are activated, and the resource table of Question 3.

For each transition $M1 \rightarrow M2$, $M2 \rightarrow M3$, and $M3 \rightarrow M4$, report the change in optimal objective value and briefly explain the main change in the optimal production plan. These effects are sequence-dependent: each one measures what a modeling feature adds given the features already present.

5.2 One observation of your own

Q6. Perform an additional numerical experiment of your choice — you may for instance change one resource availability, unit margin, setup cost, minimum production quantity, or maximum sales quantity. Report what you changed, the resulting change in the optimal solution, and a short interpretation of what you observe.

6 Deliverable

The deliverable is a single notebook, or equivalent code, containing in Markdown/LaTeX the LP formulation followed by the variables, objective terms, and constraints that the MILP adds.

All numerical data should appear clearly in one identifiable part of the notebook, rather than as constants scattered through the constraint definitions. Changing one item of data, adding or removing one constraint, and solving the resulting model again should each be straightforward. No particular software architecture is required, and simplicity and readability are preferred to unnecessary abstraction. The code is used live during the oral assessment, so it must be easy to modify by hand.

The results of Question 3, of the four models, and of your own experiment should be presented in compact form.

7 Individual oral assessment

The oral assessment takes place on **20 October 2026**. It is individual, uses the submitted notebook, and is the only basis for the grade. There is no presentation of the homework.

At the beginning of the assessment, you will receive three modifications of the production problem, of increasing difficulty. A modification may be, for instance, a change of a numerical parameter, an additional production rule, or a change in the cost or margin structure; these are examples rather than an exhaustive list.

You will first have approximately 10 minutes, without using the computer, to determine how the mathematical model should be modified in each case. You will then open your notebook and have approximately 5 minutes to implement the modifications, solve the modified problems, and report the results.