Priority option: the value of being a leader in complete and incomplete markets

M. R. Grasselli, V. Leclère, M. Ludkovski

August 22, 2012

Contents

Presentation and notation

- We consider two firms that can invest in an uncertain project by paying a fixed sunk cost of K.
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Market completeness

- We induce market completeness by assuming that (Y_t) is perfectly correlated with a traded asset P following

$$\frac{dP_t}{P_t} = \mu dt + \sigma dW_t = r dt + \sigma dW_t^{\mathbb{Q}}, \tag{2}$$

- where $W_t^{\mathbb{Q}} = W_t + \lambda t$ is a Brownian motion under the unique risk–neutral measure $\mathbb{Q}$ and $\lambda = (\mu - r)/\sigma$ is the Sharpe ratio for the asset (P_t).
- Then the dynamics of Y under $\mathbb{Q}$ is

$$\frac{dY_t}{Y_t} = (\nu - \eta\lambda)dt + \eta dW_t^{\mathbb{Q}} = (r + \eta(\xi - \lambda))dt + \eta dW_t^{\mathbb{Q}}, \tag{3}$$

where $\xi = (\nu - r)/\eta$ plays the role of a Sharpe ratio for the project.

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Valuating cash-flows

- By market completness we can use r as a discount rate to induce present value from a stream of future cash–flows.
- For example, when both firms have already invested, we find the value at time t of all future cash–flows as

$$V^F(y) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\infty} e^{-r(s-t)} D_2 Y_s^{t,y} ds\right] = \frac{D_2 y}{\underbrace{\eta(\lambda-\xi)}_{\delta}}. \quad (4)$$

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Valuating the option

- We denote a firm by L if it is the first to invest, by F if it is the second to invest and by S if both firms invest simultaneously.
- Assuming that one of the firms has already invested, the remaining firm has an option to invest in the project at a random time τ by paying the fixed cost K and start receiving cash–flows with present value given by $V^F(Y_\tau) = D_2 Y_\tau/\delta$.
- Its value is given by

$$F(y) = \sup_{\tau \in \mathcal{T}} \mathbb{E}^{\mathbb{Q}} \left[e^{-r\tau} \left(\frac{D_2 Y_\tau^{0,y}}{\delta} - K \right)^+ \mathbf{1}_{\{\tau < \infty\}} \right], \qquad (5)$$

where $\mathcal{T} := \mathcal{T}_{[0,\infty]}$ denotes the collection of all $\mathbb{F}$-stopping times with values in $[0, \infty]$.

Solving the optimization problem

- Dynamic Programming equation lead to the variational inequality

$$\min\left(rF - \frac{\eta^2}{2}y^2F'' - (r-\delta)yF', F - \left(\frac{D_2 y}{\delta} - K\right)^+\right) = 0 \quad (6)$$

supplemented by the conditions $F(v) \geq 0$ and $F(0) = 0$.

- Since the obstacle function

$$g(y) = \left(\frac{D_2 y}{\delta} - K\right)^+$$

has polynomial growth, we can use a classical verification argument to show that a candidate solution to (6) is indeed the value function $F(y)$ in (5).

Proposition

Provided $\delta = \eta(\lambda - \xi) > 0$, *the value of being the Follower at a demand level y is*

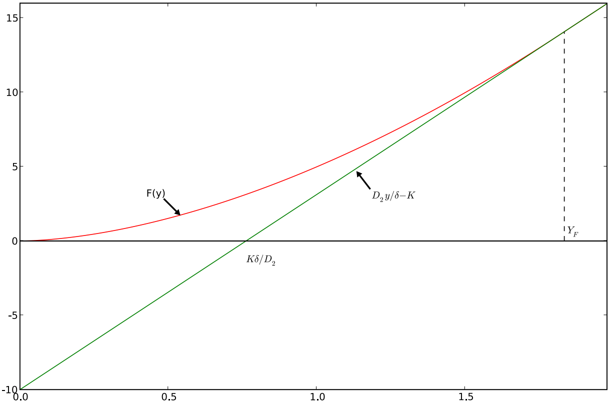
$$F(y) = \begin{cases} \frac{K}{\beta - 1}\left(\frac{y}{Y_F}\right)^{\beta} & \text{if } y \leq Y_F; \\ \frac{D_2 y}{\delta} - K & \text{if } y > Y_F, \end{cases} \tag{7}$$

where Y_F *is a threshold given by*

$$Y_F = \frac{\delta K \beta}{D_2(\beta - 1)}, \tag{8}$$

$$\beta := \left(\frac{1}{2} - \frac{r - \delta}{\eta^2}\right) + \sqrt{\left(\frac{1}{2} - \frac{r - \delta}{\eta^2}\right)^2 + \frac{2r}{\eta^2}} > 1. \tag{9}$$

Follower value



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After exercising the investment option, the leader has no further decisions to take. Thus if $y > Y_F$, it is optimal for the follower to exercise at time t and the project will have the value $D_2 y/\delta$. On the other hand, if $y \leq Y_F$ the follower will wait to invest until

$$\tau_F(y) = \inf\{s \geq t : Y_s^{t,y} = Y_F\} \tag{10}$$

and the project will have value

$$V^L(y) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\tau_F} e^{-r(s-t)} D_1 Y_s^{t,y} ds + \int_{\tau_F}^{\infty} e^{-r(s-t)} D_2 Y_s^{t,y} ds\right]$$

Proposition

Provided $\delta = \eta(\lambda - \xi) > 0$, *the value of becoming a leader at a demand level* y *is*

$$L(y) = \begin{cases} \dfrac{D_1 y}{\delta} - \dfrac{(D_1 - D_2)}{D_2} \dfrac{K\beta}{\beta - 1} \left(\dfrac{y}{Y_F}\right)^{\beta} - K & \text{if } y \leq Y_F; \\ \dfrac{D_2 y}{\delta} - K & \text{if } y > Y_F. \end{cases}$$

Moreover, the value obtained by both firms from simultaneous exercise at a demand level y is

$$S(y) = \frac{D_2 y}{\delta} - K.$$

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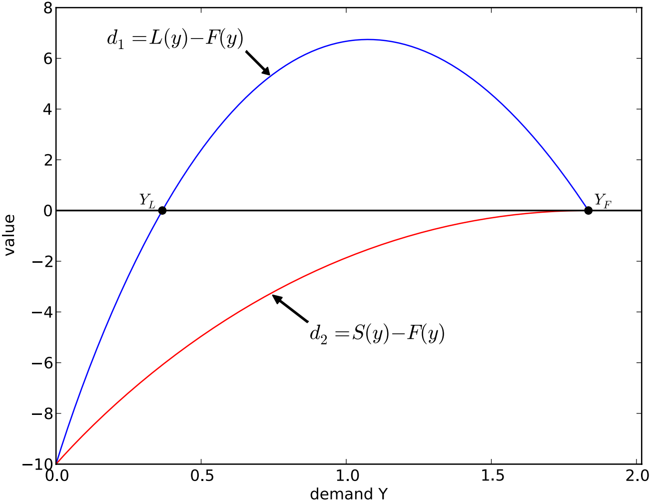

$d_1 = L(y) - F(y)$
$d_2 = S(y) - F(y)$
Y_L
Y_F
value
demand Y
8
6
4
2
0
−2
−4
−6
−8
−10
0.0
0.5
1.0
1.5
2.0

A game between Y_L and Y_F

The interesting region is $Y_L \leq y \leq Y_F$, where each firm prefers to be the leader, but at the same time both firms are worse off by investing simultaneously than being the follower. This is precisely the situation where the coordination game is played, with the z-th round at "time" $(0, z)$ having payoffs

	Invest	Wait
Invest	$(S(y), S(y))$	$(F(y), L(y))$
Wait	$(L(y), F(y))$	Repeat

Given a mixed strategy $(p_1(y), p_2(y))$ with $\max(p_1(y), p_2(y)) > 0$, at least one firm will immediately exercise a.s., and the probabilities of the three possible outcomes are

$$a_1(y) = \frac{p_1(y)(1 - p_2(y))}{p_1(y) + p_2(y) - p_1(y)p_2(y)} \qquad \text{(firm 1 exercises)},$$

$$a_2(y) = \frac{(1 - p_1(y))p_2(y)}{p_1(y) + p_2(y) - p_1(y)p_2(y)} \qquad \text{(firm 2 exercises)},$$

$$a_S(y) = \frac{p_1(y)p_2(y)}{p_1(y) + p_2(y) - p_1(y)p_2(y)} \qquad \text{(simultaneous exercise)}.$$

Thus, the expected payoff for firm 1 when $Y_L < y < Y_F$ is

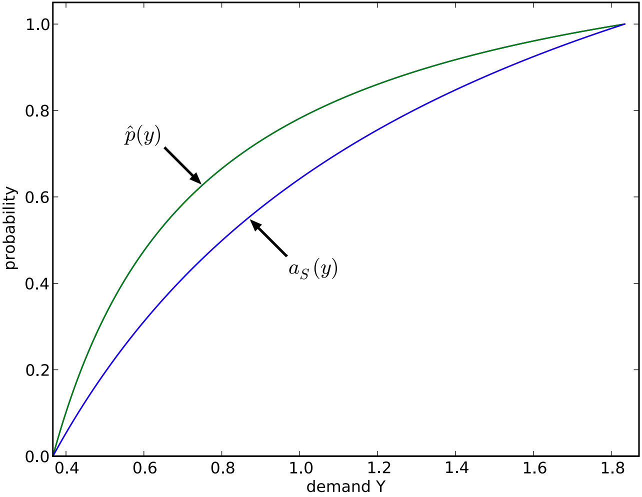
$$E_1\left(y; p_1, p_2\right) = a_1(y)L\left(y\right) + a_2(y)F\left(y\right) + a_S(y)S\left(y\right), \qquad (11)$$

and similarly for firm 2. Maximizing (11) with respect to p_1 subject to $p_1 = p_2$, we find that the unique symmetric Nash equilibrium of the stage game is given by

$$\widehat{p}(y) = \frac{L(y) - F(y)}{L(y) - S(y)}, \qquad Y_L < y < Y_F. \qquad (12)$$

It follows that the probability of simultaneous investment for a given demand $y \in (Y_L, Y_F)$ is

$$a_S(y) = \frac{L(y) - F(y)}{L(y) + F(y) - 2S(y)}, \qquad (13)$$


1.0
0.8
0.6
0.4
0.2
0.0
probability
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
demand Y
$\hat{p}(y)$
$a_S(y)$

Theorem

There exists a symmetric, Markov, sub–game perfect equilibrium with strategies depending on the level of demand as follows:

1. *If $y < Y_L$, both firms wait for the demand to reach Y_L.*
2. *At $y = Y_L$, there is no simultaneous exercise and each firm has an equal probability of emerging as a leader while the other becomes a follower and waits until demand rises to Y_F.*
3. *If $Y_L < y < Y_F$, each firm chooses a mixed strategy consisting of exercising the option to invest with probability $\widehat{p}(y)$. The resulting equilibrium yields simultaneous exercise with probability $a_S(y)$ given in (13) and the case where one firm emerges as the leader and the other waits until demand rises to Y_F with probability $(1 - a_S(y))$.*
4. *If $y \geq Y_F$, both firms invest immediately.*

We can note that the optimal probability $\widehat{p}$ corresponds to the probability that makes each firm indifferent between being the follower or playing the game described above, which is a restatement of the concept of rent equalization from Fudenberg and Tirole that implies that in equilibrium the timing value of the leader option completely vanishes due to strategic preemption. Thus

$$F(y) = E_1\left(y; \hat{p}, \hat{p}\right) = E_2\left(y; \hat{p}, \hat{p}\right)$$

Contents

Definition

- We have assumed so far that the roles of leader and follower are not predetermined.
- Alternatively, we could consider a Stackelberg game where the roles of the firms are predetermined exogenously.
- We are valuating this advantage, and calling it a priority option.

The leader has the option to invest in the project knowing that the follower is forbidden to invest until the leader has done so. That is, the leader can invest in the project at a random time τ and receive the payoff $L(Y_\tau)$. Therefore, the value function for the leader in this case is

$$L^\pi(y) = \sup_{\tau \in \mathcal{T}} \mathbb{E}^{\mathbb{Q}} \left[e^{-r\tau} L(Y_\tau^{0,y})^+ \mathbf{1}_{\{\tau < \infty\}} \right], \qquad (14)$$

where the superscript π is meant to indicate that the leader now has the priority to invest. As before, the dynamic programming equation associated with this optimal stopping problem is

$$\min \left(rL^\pi - \frac{\eta^2}{2} y^2 (L^\pi)'' - (r-\delta) y (L^\pi)', L^\pi - L^+ \right) = 0. \qquad (15)$$

Let us define the constants Y_1 and A_1 as

$$Y_1 := \frac{\delta K \beta}{D_1(\beta - 1)}; \tag{16}$$

$$A_1 := \frac{1}{Y_1^{\beta}} \frac{K}{\beta - 1} \left(\frac{D_2}{D_1}\right)^{\beta} \left[\left(\frac{D_1}{D_2}\right)^{\beta} - \beta\left(\frac{D_1}{D_2}\right) + \beta\right], \tag{17}$$

and the constants Y_2, Y_3, A_2, A_3 as a solution of the nonlinear system of equations

$$\begin{cases} A_2 Y_2^{\beta} + A_3 Y_2^{\beta_1} = \dfrac{D_1 Y_2}{\delta} - \dfrac{(D_1 - D_2) Y_F}{\delta} \left(\dfrac{Y_2}{Y_F}\right)^{\beta} - K; \\ \beta A_2 Y_2^{\beta - 1} + \beta_1 A_3 Y_2^{\beta_1 - 1} = \dfrac{D_1 Y_2}{\delta} - \dfrac{(D_1 - D_2)\beta Y_F}{\delta} \left(\dfrac{Y_2}{Y_F}\right)^{\beta}; \\ A_2 Y_3^{\beta} + A_3 Y_3^{\beta_1} = \dfrac{D_2 Y_3}{\delta} - K; \\ \beta A_2 Y_3^{\beta - 1} + \beta_1 A_3 Y_3^{\beta_1 - 1} = \dfrac{D_2 Y_3}{\delta}. \end{cases}$$

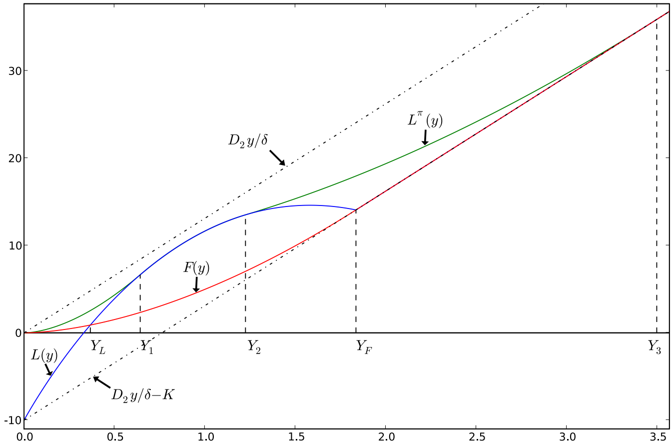
Theorem

Let Y_1 *and* A_1 *be given by* (16) *and* (17) *and assume that the nonlinear system* (18) *has a unique solution given by the constants* Y_2, Y_3, A_2, A_3. *If*

$$0 < Y_1 < Y_2 < Y_F < Y_3, \tag{19}$$

then the solution to (14) *is given by*

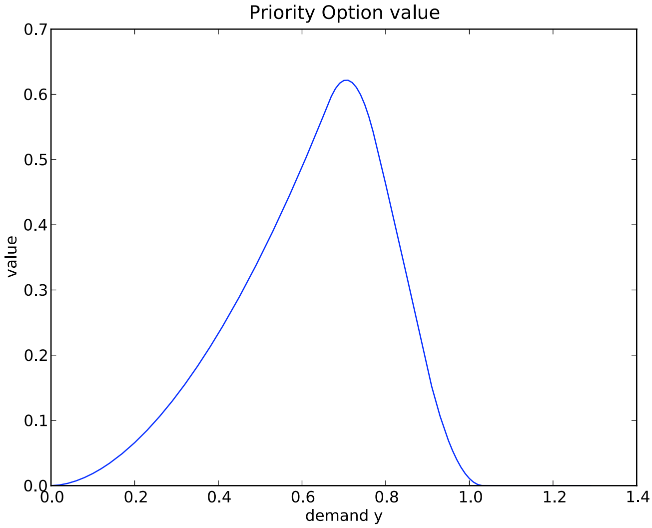
$$L^\pi(y) = \begin{cases} A_1 y^\beta & \text{if } 0 \leq y < Y_1; \\ L(y) & \text{if } Y_1 \leq y \leq Y_2; \\ A_2 y^\beta + A_3 y^{\beta_1} & \text{if } Y_2 < y < Y_3; \\ L(y) & \text{if } y \geq Y_3, \end{cases} \tag{20}$$


D_2y/δ
$L^\pi(y)$
$F(y)$
$L(y)$
$D_2y/\delta - K$
Y_L
Y_1
Y_2
Y_F
Y_3
30
20
10
0
-10
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5

Proposition

The value of $\pi(y) = L^\pi(y) - F(y)$ *of the priority option is given by*

$$\pi(y) = \begin{cases} \left[\left(\frac{D_1}{D_2}\right)^\beta - \beta\left(\frac{D_1}{D_2}\right) + \beta - 1\right]\frac{K}{\beta-1}\left(\frac{y}{Y_F}\right)^\beta & 0 \le y < Y_1 \\ \frac{D_1}{\delta}y - K - \left[\beta\left(\frac{D_1}{D_2}\right) - \beta + 1\right]\frac{K}{\beta-1}\left(\frac{y}{Y_F}\right)^\beta & Y_1 \le y < Y_2 \\ \left(A_2 Y_F^\beta - \frac{K}{\beta-1}\right)\left(\frac{y}{Y_F}\right)^\beta + A_3 y^{\beta_1} & Y_2 \le y < Y_F \\ A_2 y^\beta + A_3 y^{\beta_1} - \frac{D_2 y}{\delta} + K & Y_F \le y < Y_3 \\ 0 & Y_3 \le y \end{cases}$$


Priority Option value
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
value
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
demand y

Lemma

$$F^{\pi}(y) = \begin{cases} F(Y_1)\left(\frac{y}{Y_1}\right)^{\beta} \\ \frac{K}{\beta-1}\left(\frac{y}{Y_F}\right)^{\beta} \\ F(Y_2)\left(\frac{y}{Y_2}\right)^{A}\frac{\left(\frac{Y_3}{y}\right)^{B}-\left(\frac{Y_3}{y}\right)^{-B}}{\left(\frac{Y_3}{Y_2}\right)^{B}-\left(\frac{Y_3}{Y_2}\right)^{-B}} + F(Y_3)\left(\frac{y}{Y_3}\right)^{A}\frac{\left(\frac{y}{Y_2}\right)^{B}-\left(\frac{y}{Y_2}\right)^{-B}}{\left(\frac{Y_3}{Y_2}\right)^{B}-\left(\frac{Y_3}{Y_2}\right)^{-B}} \\ \frac{D_2 y}{\delta} - K \end{cases}$$

$$\begin{aligned} A &= \frac{1}{2} - \frac{r-\delta}{\eta^2}, \\ B &= \sqrt{A^2 + \frac{2r}{\eta^2}} = \beta - A. \end{aligned}$$

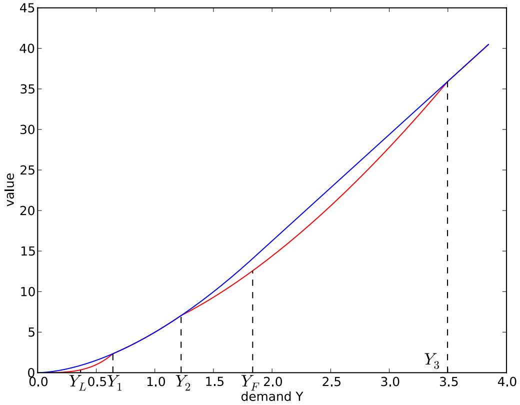


Figure: Value F^{π} for a predetermined follower compared with the follower value F obtained when roles are not predetermined.

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- We drop the assumption of a complete market.
- Indeed, many real options involve non-traded underlying assets, such as real estate prices, pharmaceutical developments, etc.
- In incomplete markets it is delicate to go from cash–flows to project value, since cash–flows received at different times cannot be easily compared.
- Here we treat project values as lump-sum payoffs instead of present values of future cash–flows and compare payoffs at different times using certainty equivalent arguments in the context of optimal utility of terminal wealth.

Accordingly, the project value V_t is now assumed to be partially correlated with a traded asset P_t as follows:

$$\frac{dV_t}{V_t} = \nu dt + \eta(\rho dW_t + \sqrt{1-\rho^2} dW_t^0), \tag{21}$$

$$\frac{dP_t}{P_t} = \mu dt + \sigma dW_t, \tag{22}$$

where $\rho \in (-1, 1)$ is a constant and (W_t^0) is a Brownian motion independent of (W_t). Observe that the dynamics takes place under the physical measure $\mathbb{P}$ and that (21) reduces to a complete market in the limit $\rho \to 1$.

- For a monopolistic firm, investing in the project at time t means receiving a lump sum equal to V_t.
- In the duopoly case considered here, if a firm invests after another firm has already invested it receives a reduced lump sum equal to $(1-a)V_t$, for some $0 < a < 1$, whereas the other firm keeps a fraction bV_t of the original project value, with $0 < b < 1$.
- Setting $b = (1-a)$ is analogous to the framework used before.
- Next we assume that both firms act as utility maximizing agents with an exponential utility function $U(x) = -e^{-\gamma x}$, where $\gamma > 0$ is the risk aversion coefficient.
- In addition to investing in the project, the firms can allocate an amount θ_t to be invested at time t in the traded asset with price P_t.

$$dX_t^\theta = \theta_t \frac{dP_t}{P_t} = \theta_t \sigma(\lambda dt + dW_t). \tag{23}$$

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Define the value function for the follower (following V.Henderson argument) as

$$f(x,v) = \sup_{(\tau,\theta)} \mathbb{E}\left[e^{\frac{\lambda^2 \tau}{2}} U\left(X_\tau^{0,x,\theta} + \left((1-a) V_\tau^{0,v} - K \right)^+ \right) \right]. \quad (24)$$

The choice of the discount factor $e^{\frac{\lambda^2 \tau}{2}}$ leads to a horizon unbiased optimization problem.

if we set

$$\beta \equiv \beta(\rho) := 1 + \frac{2(\rho\lambda - \xi)}{\eta} > 1, \tag{25}$$

and define $V_F = V^*/(1-a)$, where $V^* \equiv V^*(\rho)$ is the solution to the nonlinear equation

$$\kappa(V^* - K) = \log\left[1 + \frac{\kappa V^*}{\beta}\right], \qquad \kappa := \gamma(1-\rho^2), \tag{26}$$

then $(1-a)V_F > K$ and the follower value function is given by $f(x,v) = -e^{-\gamma(x+F(v))}$, where

$$F(v) = \begin{cases} -\frac{1}{\kappa}\log\left[1 - \left(1 - e^{-\kappa((1-a)V_F - K)}\right)\left(\frac{v}{V_F}\right)^{\beta}\right] & \text{if } 0 \le v \le V_F; \\ (1-a)v - K & \text{if } v > V_F. \end{cases} \tag{27}$$

- We represent the reduction in project value experienced by the leader upon entrance of the follower as a lump sum loss $(1-b)V_{\tau_F}^{t,v}$ at τ_F and consider its utility indifference value for the leader at time t.
- utility of Leader

$$h(x,v) = \sup_{\theta} \mathbb{E}\left[e^{\frac{\lambda^2\tau}{2}} U\left(X_{\tau_F}^{0,x,\theta} - (1-b)V_{\tau_F}^{0,v}\right)\right], \qquad (28)$$

- therefore,

$$h(x,v) = U(x)\left[1 - \left(1 - e^{\kappa(1-b)V_F}\right)\left(\frac{v}{V_F}\right)^{\beta}\right]^{\frac{1}{1-\rho^2}}, \qquad (29)$$

- We then define the utility indifference value $H_F(v)$ for the reduction in project value experienced by the leader through the equality

$$h(x - H_F(v), 0) = h(x, v), \tag{30}$$

- from which it follows that

$$H_F(v) = \frac{1}{\kappa} \log \left[1 - \left(1 - e^{\kappa(1-b)V_F} \right) \left(\frac{v}{V_F} \right)^{\beta} \right]. \tag{31}$$

- Using $H_F(v)$, we can incorporate the expected reduction in project value at τ_F into the value function $\ell(x, v) = -e^{-\gamma(x+L(v))}$ for the leader simply by setting

$$L(v) = \begin{cases} v - H_F(v) - K & \text{if } v \le V_F; \\ bv - K & \text{if } v > V_F. \end{cases} \tag{32}$$

With priority

Define V_1 as the solution of

$$\kappa L(V_1) = \log\left[1 + \frac{\kappa}{\beta} V_1 L'(V_1)\right]. \tag{33}$$

$$B_1 := (1 - e^{-\kappa L(V_1)}) V_1^{-\beta}$$

Next define the constants V_2, V_3, B_2, B_3 as a solution to the nonlinear system

$$\begin{aligned} B_2 + B_3 V_2^{\beta} &= -e^{-\kappa L(V_2)}; \\ \beta B_3 V_2^{\beta_1 - 1} &= \kappa e^{-\kappa L(V_2)} L'(V_2); \\ B_2 + B_3 V_3^{\beta} &= -e^{-\kappa L(V_3)}; \\ \beta B_3 V_3^{\beta_1 - 1} &= \kappa e^{-\kappa L(V_3)} L'(V_3). \end{aligned} \tag{34}$$

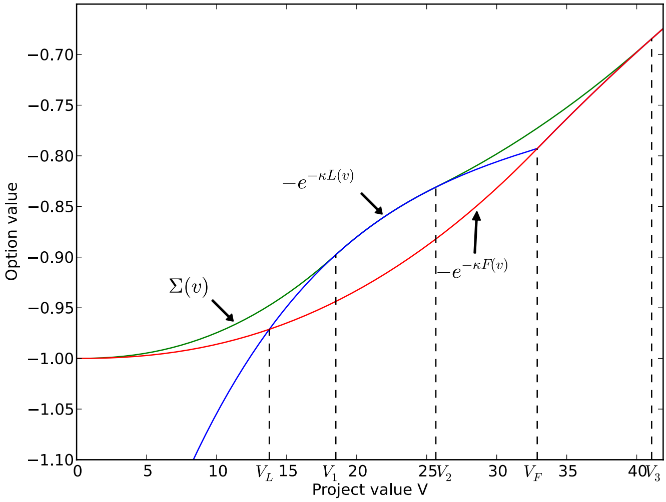
Theorem

Let V_1 and B_1 be given and assume that the nonlinear system has a solution (V_2, V_3, B_2, B_3). If

$$0 < V_1 < V_2 < V_F < V_3, \tag{35}$$

then

$$\Sigma(v) = \begin{cases} -1 + B_1 v^{\beta} & \text{if } 0 \leq v < V_1; \\ -e^{-\kappa L(v)} & \text{if } V_1 \leq v \leq V_2; \\ B_2 + B_3 v^{\beta} & \text{if } V_2 < v < V_3; \\ -e^{-\kappa L(v)} & \text{if } v \geq V_3. \end{cases} \tag{36}$$


−0.70
−0.75
−0.80
−0.85
−0.90
−0.95
−1.00
−1.05
−1.10
Option value
$-e^{-\kappa L(v)}$
$\Sigma(v)$
$-e^{-\kappa F(v)}$
0
5
10
V_L
15
V_1
20
25
V_2
30
V_F
35
40
V_3
Project value V

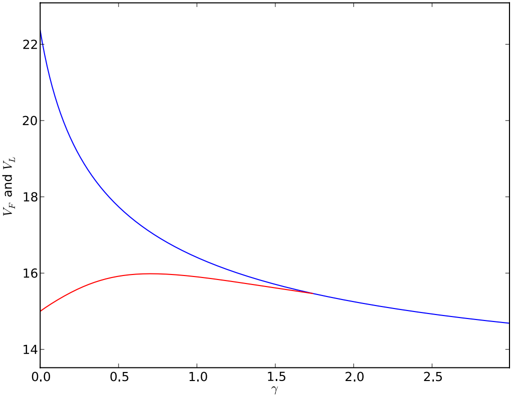


Figure: Leader and Follower investment thresholds in the incomplete market case, when $a = 0.2$. Parameters are the same as Figure 1, with $\rho = 0.8$.

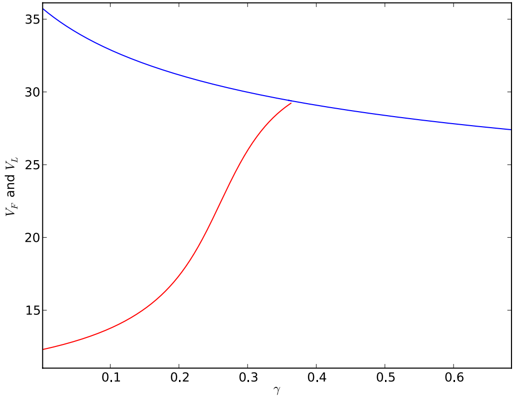


Figure: Leader and Follower investment thresholds in the incomplete market case, when $a = 0.5$. Parameters are the same as Figure 1, with $\rho = 0.8$.

Conclusion

- We can model and price the value of being the leader and the follower in a duopoly for some investment project.
- The pricing has been done as well in a complete and an incomplete market.
- To determine who is going to be the leader we propose an instantaneous game and find the optimal markovian symmetric subgame equilibrium.
- From that point we value the advantage of having the priority, and being sure to be the first to invest.
- The priority option will retard any investment, and create a new range of demand ($[Y_2, Y_3]$) where there is no investment.

Thank you for your attention, questions ?