Spatial Decomposition in Stochastic Optimization: Theoretical and Practical Questions.

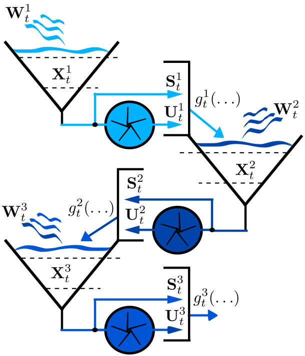
P. Carpentier, J-Ph. Chancelier, M. De Lara, V. Leclère

SESO Week

2014, June 24

Mulststage Stochastic Optimization : an Example

How to manage a chain of dam producing electricity from the turbine water to optimize the gain ?



$$\mathbb{E}\left[\sum_{i=1}^{N}\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K_i(\mathbf{X}_T^i)\right]$$

Constraints:

- dynamics: $\mathbf{X}_{t+1} = f_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1})$,
- nonanticipativity: $\mathbf{U}_t \preceq \mathcal{F}_t$,
- spatial coupling: $\mathbf{Z}_t^{i+1} = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i)$.

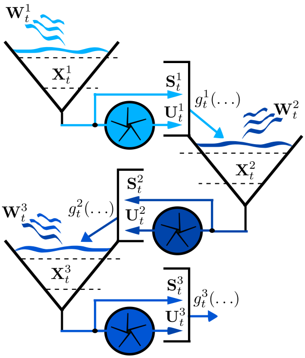
Mulststage Stochastic Optimization : an Example

How to manage a chain of dam producing electricity from the turbine water to optimize the gain ?

$$\mathbb{E}\left[\sum_{i=1}^{N}\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i,\mathbf{U}_t^i,\mathbf{W}_{t+1})+K_i(\mathbf{X}_T^i)\right]$$

Constraints:

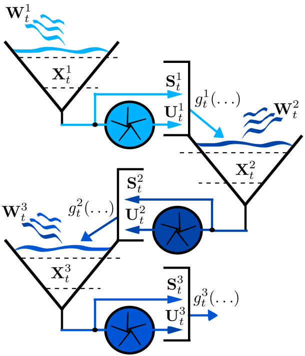
- dynamics:
  $\mathbf{X}_{t+1} = f_t(\mathbf{X}_t,\mathbf{U}_t,\mathbf{W}_{t+1})$,
- nonanticipativity:
  $\mathbf{U}_t \preceq \mathcal{F}_t$,
- spatial coupling:
  $\mathbf{Z}_t^{i+1} = g_t^i(\mathbf{X}_t^i,\mathbf{U}_t^i,\mathbf{W}_{t+1}^i)$.



Mulststage Stochastic Optimization : an Example

How to manage a chain of dam producing electricity from the turbine water to optimize the gain ?

$$\mathbb{E}\left[\sum_{i=1}^{N}\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K_i(\mathbf{X}_T^i)\right]$$



Constraints:

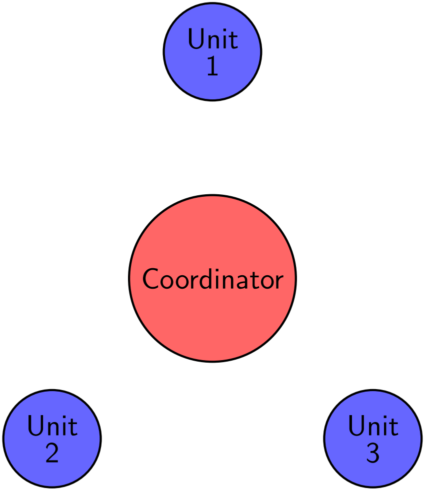
- dynamics:
  $\mathbf{X}_{t+1} = f_t(\mathbf{X}_t, \mathbf{U}_t, \mathbf{W}_{t+1})$,
- nonanticipativity:
  $\mathbf{U}_t \preceq \mathcal{F}_t$,
- spatial coupling:
  $\mathbf{Z}_t^{i+1} = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i)$.

Contents

Presentation Outline

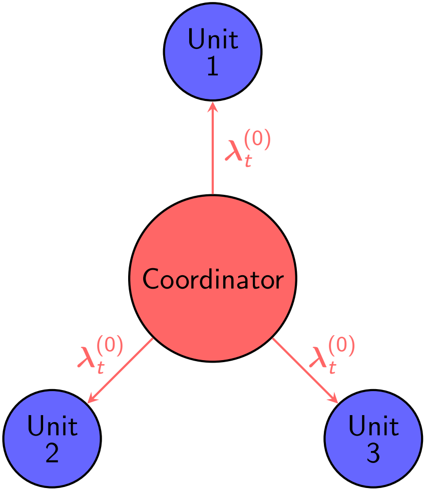
Intuition of Spatial Decomposition

- Satisfy a demand with N units of production at minimal cost.
- Price decomposition:
  - the coordinator sets a sequence of price λ_t,
  - the units send their production planning $\mathbf{U}_t^{(i)}$,
  - the coordinator compare total production and demand and updates the price,
  - and so on...



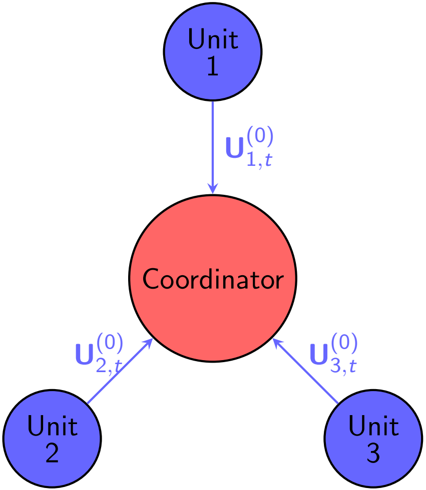
Intuition of Spatial Decomposition

- Satisfy a demand with N units of production at minimal cost.
- Price decomposition:
    - the coordinator sets a sequence of price λ_t,
    - the units send their production planning $\mathbf{U}_t^{(i)}$,
    - the coordinator compare total production and demand and updates the price,
    - and so on...



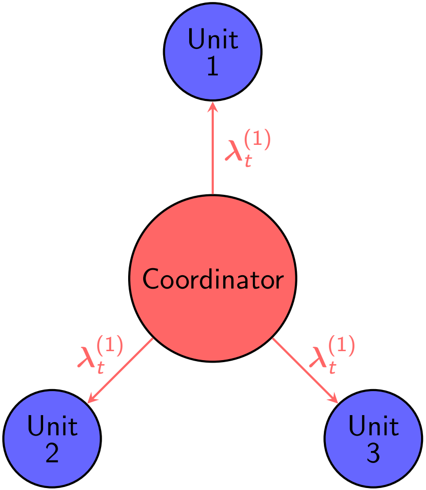
Intuition of Spatial Decomposition

- Satisfy a demand with N units of production at minimal cost.
- Price decomposition:
  - the coordinator sets a sequence of price λ_t,
  - the units send their production planning $\mathbf{U}_t^{(i)}$,
  - the coordinator compare total production and demand and updates the price,
  - and so on...



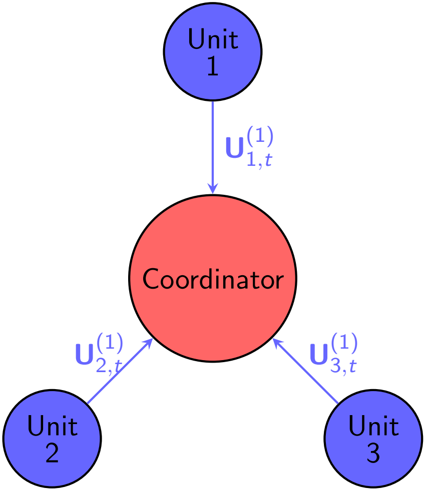
Intuition of Spatial Decomposition

- Satisfy a demand with N units of production at minimal cost.
- Price decomposition:
  - the coordinator sets a sequence of price λ_t,
  - the units send their production planning $\mathbf{U}_t^{(i)}$,
  - the coordinator compare total production and demand and updates the price,
  - and so on...



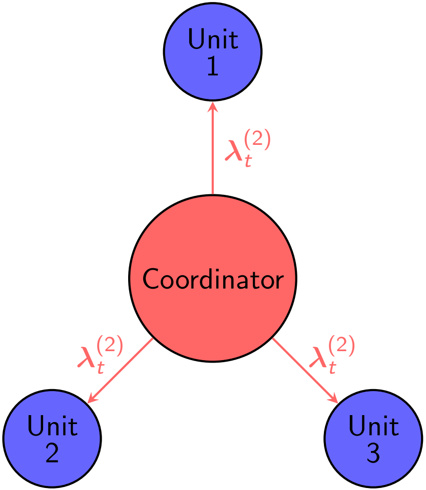
Intuition of Spatial Decomposition

- Satisfy a demand with N units of production at minimal cost.
- Price decomposition:
  - the coordinator sets a sequence of price λ_t,
  - the units send their production planning $\mathbf{U}_t^{(i)}$,
  - the coordinator compare total production and demand and updates the price,
  - and so on...



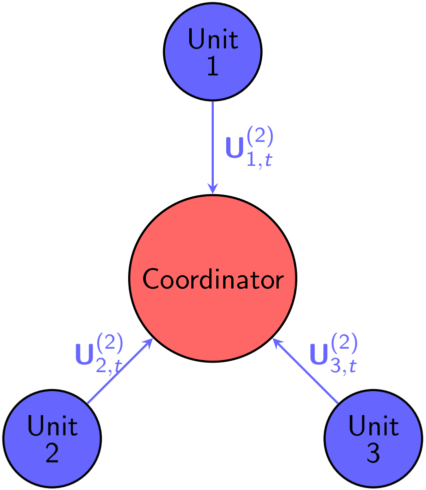
Intuition of Spatial Decomposition

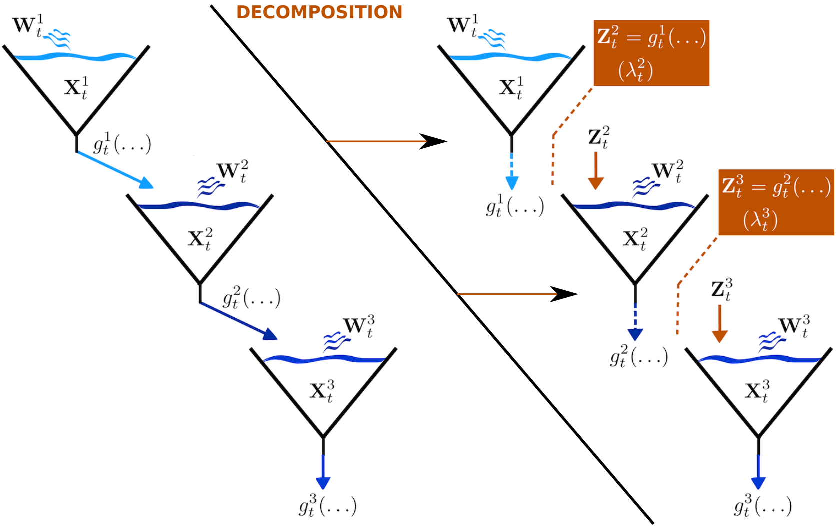
- Satisfy a demand with N units of production at minimal cost.
- Price decomposition:
  - the coordinator sets a sequence of price λ_t,
  - the units send their production planning $\mathbf{U}_t^{(i)}$,
  - the coordinator compare total production and demand and updates the price,
  - and so on...



Intuition of Spatial Decomposition

- Satisfy a demand with N units of production at minimal cost.
- Price decomposition:
    - the coordinator sets a sequence of price λ_t,
    - the units send their production planning $\mathbf{U}_t^{(i)}$,
    - the coordinator compare total production and demand and updates the price,
    - and so on...




DECOMPOSITION
$\mathbf{W}_t^1$
$\mathbf{X}_t^1$
$g_t^1(\dots)$
$\mathbf{W}_t^2$
$\mathbf{X}_t^2$
$g_t^2(\dots)$
$\mathbf{W}_t^3$
$\mathbf{X}_t^3$
$g_t^3(\dots)$
$\mathbf{Z}_t^2 = g_t^1(\dots)$
(λ_t^2)
$\mathbf{Z}_t^2$
$\mathbf{Z}_t^3 = g_t^2(\dots)$
(λ_t^3)
$\mathbf{Z}_t^3$

Presentation Outline

Primal Problem

$$\min_{\mathbf{X},\mathbf{U}} \sum_{i=1}^{N} \mathbb{E}\Big[\sum_{t=0}^{T} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i)\Big]$$

$$\forall\, i, \quad \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}), \quad \mathbf{X}_0^i = x_0^i,$$

$$\forall\, i, \quad \mathbf{U}_t^i \in \mathcal{U}_{t,i}^{ad}, \quad \mathbf{U}_t^i \preceq \mathcal{F}_t,$$

$$\sum_{i=1}^{N} \theta_t^i(\mathbf{U}_t^i) = 0$$

Solvable by DP with state $(\mathbf{X}_1, \dots, \mathbf{X}_N)$

Primal Problem

$$\min_{\mathbf{X},\mathbf{U}} \sum_{i=1}^{N} \mathbb{E}\Big[\sum_{t=0}^{T} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i)\Big]$$

$$\forall\, i, \quad \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}), \quad \mathbf{X}_0^i = x_0^i,$$

$$\forall\, i, \quad \mathbf{U}_t^i \in \mathcal{U}_{t,i}^{ad}, \quad \mathbf{U}_t^i \preceq \mathcal{F}_t,$$

$$\sum_{i=1}^{N} \theta_t^i(\mathbf{U}_t^i) = 0 \qquad \rightsquigarrow \boldsymbol{\lambda}_t \quad \text{multiplier}$$

Solvable by DP with state $(\mathbf{X}_1, \dots, \mathbf{X}_N)$

Primal Problem with Dualized Constraint

$$\min_{\mathbf{X},\mathbf{U}} \max_{\boldsymbol{\lambda}} \sum_{i=1}^{N} \mathbb{E}\Big[\sum_{t=0}^{T} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + \langle \boldsymbol{\lambda}_t, \theta_t^i(\mathbf{U}_t^i)\rangle + K^i(\mathbf{X}_T^i)\Big]$$

$$\forall\, i, \quad \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}), \quad \mathbf{X}_0^i = x_0^i,$$

$$\forall\, i, \quad \mathbf{U}_t^i \in \mathcal{U}_{t,i}^{ad}, \quad \mathbf{U}_t^i \preceq \mathcal{F}_t,$$

Coupling constraint dualized $\Longrightarrow$ all constraints are unit by unit

Dual Problem

$$\max_{\boldsymbol{\lambda}} \min_{\mathbf{X},\mathbf{U}} \sum_{i=1}^{N} \mathbb{E}\Big[\sum_{t=0}^{T} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + \langle \boldsymbol{\lambda}_t, \theta_t^i(\mathbf{U}_t^i)\rangle + K^i(\mathbf{X}_T^i)\Big]$$

$$\forall\, i, \quad \mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}), \quad \mathbf{X}_0^i = x_0^i,$$

$$\forall\, i, \quad \mathbf{U}_t^i \in \mathcal{U}_{t,i}^{ad}, \quad \mathbf{U}_t^i \preceq \mathcal{F}_t,$$

Exchange operator min and max to obtain a new problem

Decomposed Dual Problem

$$\max_{\lambda} \sum_{i=1}^{N} \min_{\mathbf{X}^i,\mathbf{U}^i} \mathbb{E}\Big[\sum_{t=0}^{T} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + \langle \boldsymbol{\lambda}_t, \theta_t^i(\mathbf{U}_t^i)\rangle + K^i(\mathbf{X}_T^i)\Big]$$

$$\mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}), \quad \mathbf{X}_0^i = x_0^i,$$

$$\mathbf{U}_t^i \in \mathcal{U}_{t,i}^{ad}, \quad \mathbf{U}_t^i \preceq \mathcal{F}_t,$$

For a given λ, minimum of sum is sum of minima

Inner Minimization Problem

$$\min_{\mathbf{X}^i,\mathbf{U}^i} \mathbb{E}\Big[\sum_{t=0}^{T} L_t^i(\mathbf{X}_t^i,\mathbf{U}_t^i,\mathbf{W}_{t+1}) + \langle \boldsymbol{\lambda}_t, \theta_t^i(\mathbf{U}_t^i)\rangle + K^i(\mathbf{X}_T^i)\Big]$$

$$\mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i,\mathbf{U}_t^i,\mathbf{W}_{t+1}), \quad \mathbf{X}_0^i = x_0^i,$$

$$\mathbf{U}_t^i \in \mathcal{U}_{t,i}^{ad}, \quad \mathbf{U}_t^i \preceq \mathcal{F}_t,$$

We have N smaller subproblems. Can they be solved by DP ?

Inner Minimization Problem

$$\min_{\mathbf{X}^i,\mathbf{U}^i} \mathbb{E}\left[\sum_{t=0}^{T} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + \langle \boldsymbol{\lambda}_t, \theta_t^i(\mathbf{U}_t^i)\rangle + K^i(\mathbf{X}_T^i)\right]$$

$$\mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}), \quad \mathbf{X}_0^i = x_0^i,$$

$$\mathbf{U}_t^i \in \mathcal{U}_{t,i}^{ad}, \quad \mathbf{U}_t^i \preceq \mathcal{F}_t,$$

No : $\boldsymbol{\lambda}$ is a time-dependent noise $\rightsquigarrow$ state $(\mathbf{W}_1, \dots, \mathbf{W}_t)$

A Few Questions

- In which space lives the multiplier process $\boldsymbol{\lambda}$? For which duality ?
  - L^2
  - L^1
  - $\left(\mathrm{L}^\infty\right)^\star$
- What are the relations between the primal and dual problems ?
- Can we solve the subproblems by Dynamic Programming ?
  $\rightsquigarrow$ No!
- How to update the multiplier process ?
  Uzawa Algorithm

Presentation Outline

Problem Statement

We consider the following (primal) problem:

$$(\mathcal{P}) \qquad \min_{u \in \mathcal{U}^{\mathrm{ad}}} \quad J(u) \,, \qquad s.t. \quad \Theta(u) \in -C \,.$$

Where $\mathcal{U}$ and $\mathcal{V}$ are two Hausdorff spaces, and

- $J : \mathcal{U} \to \bar{\mathbb{R}}$ is an objective function ,
- $\Theta : \mathcal{U} \to \mathcal{V}$ is a constraint function (to be dualized),
- $C \subset \mathcal{V}$ is a cone of constraints,
- $\mathcal{U}^{\mathrm{ad}} \subset \mathcal{U}$ is a constraint set (not to be dualized).

Dual Problem

The primal problem can be written

$$(\mathcal{P}) \qquad \min_{u \in \mathcal{U}^{\mathrm{ad}}} \quad \max_{\lambda \in C^\star} \qquad J(u) + \left\langle \lambda \,, \Theta(u) \right\rangle_{\mathcal{V}^\star,\mathcal{V}} ,$$

where $C^\star \subset \mathcal{V}^\star$ is given by

$$C^\star = \left\{ \lambda \in \mathcal{V}^\star \mid \forall x \in C, \quad \left\langle \lambda \,, x \right\rangle_{\mathcal{V}^\star,\mathcal{V}} \geq 0 \right\} .$$

The dual problem of Problem $(\mathcal{P})$ reads

$$(\mathcal{D}) \qquad \max_{\lambda \in C^\star} \quad \min_{u \in \mathcal{U}^{\mathrm{ad}}} \qquad J(u) + \left\langle \lambda \,, \Theta(u) \right\rangle_{\mathcal{V}^\star,\mathcal{V}} .$$

Gradient of the Dual

Assume that $\mathcal{U} = \mathcal{U}^\star$, and $\mathcal{V} = \mathcal{V}^\star$ are Hilbert spaces. Recall the dual problem $\big(\mathcal{D}\big)$ as

$$\max_{\lambda \in C^\star} \quad \underbrace{\min_{u \in \mathcal{U}^{\mathrm{ad}}} \quad \Big\{ J(u) + \big\langle \lambda \, , \Theta(u) \big\rangle_{\mathcal{V}^\star, \mathcal{V}} \Big\}}_{:=\varphi(\lambda)} \ .$$

Under some regularity and unicity conditions, if $u^\sharp(\lambda)$ is a minimizer of the above problem, then

$$\Theta\big(u^\sharp(\lambda)\big) = \nabla \varphi\big(\lambda\big) \ .$$

Uzawa Algorithm

Data: Initial multiplier $\lambda^{(0)} \in \mathcal{V}$, step $\rho > 0$;
Result: Optimal solution $u^\sharp$ and multiplier $\lambda^\sharp$;
repeat

$$u^{(k)} \in \operatorname*{arg\,min}_{u \in \mathcal{U}^{\mathrm{ad}}} \Big\{ J(u) + \big\langle \lambda^{(k)} , \Theta(u) \big\rangle \Big\} ,$$

$$\lambda^{(k+1)} = \mathrm{proj}_{C^\star} \Big(\lambda^{(k)} + \rho\, \Theta\big(u^{(k)}\big) \Big) .$$

until $\Theta(u^{(k)}) \in -C$;

Presentation Outline

L^∞ setting

From now on we consider that

$$\begin{aligned} \mathcal{U} &= \mathrm{L}^\infty\big(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{R}^n\big) \ , \\ \mathcal{V} &= \mathrm{L}^\infty\big(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{R}^m\big) \ , \\ C &= \{0\}. \end{aligned}$$

Where the σ-algebra is not finite (modulo $\mathbb{P}$). Hence, $\mathcal{U}$ and $\mathcal{V}$ are non-reflexive, non-separable, Banach spaces.
If the σ-algebra is finite modulo $\mathbb{P}$, $\mathcal{U}$ and $\mathcal{V}$ are finite dimensional spaces, and the usual result applies.

Perks of an Hilbert Space

Fact

In an Hilbert space $\mathcal{H}$ we know that

i) the weak and weak* topologies are identical,

ii) the space $\mathcal{H}$ and its topological dual can be identified.

Point *i*) allows to formulate existence of minimizer results:

- weakly closed bounded $\Longrightarrow$ weakly compact;
- for a convex set: weakly closed $\Longleftrightarrow$ closed;
- for a convex function: weakly l.s.c $\Longleftrightarrow$ l.s.c.

Hence, a coercive, l.s.c. function J admits an infimum.
Point *ii*) allows to write gradient-like algorithm: at any iteration k, linear combination of $\lambda^{(k)}$ and $g^{(k)}$ take place in $\mathcal{H}$.

Difficulties Appearing in a Banach Space

- Reflexive Banach space:
  - *i*) still holds true ($\rightsquigarrow$ existence of minimizers)
  - *ii*) no longer true ($\rightsquigarrow$ linear combination of $u^{(k)} \in E$ and $g^{(k)} \in E^\star$ does not have any sense).
- Non-reflexive Banach space E: neither *i*) nor *ii*) holds true.
- E is the topological dual of a Banach space: a weakly* closed bounded subset of E is weak* compact. Thus, weak* lower semicontinuity and coercivity of a function J gives the existence of minimizers of J.

Specificities of $\mathrm{L}^\infty(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{R}^n)$

- L^∞ is the topological dual of the Banach space L^1. Hence, if J is weak* l.s.c and coercive, then J admits a minimizer.
- L^∞ can be identified with a subset of its topological dual $\left(\mathrm{L}^\infty\right)^\star$. Thus,

$$\boldsymbol{\lambda}^{(k+1)} = \boldsymbol{\lambda}^{(k)} + \rho\, \Theta\big(\mathbf{U}^{(k)}\big) \ ,$$

  make sense: it is a linear combination of elements of $\left(\mathrm{L}^\infty\right)^\star$.
- Moreover, if $\lambda^{(0)}$ is chosen in L^∞, then the sequence $\{\boldsymbol{\lambda}^{(k)}\}_{k\in\mathbb{N}}$ remains in L^∞.

Uzawa Algorithm

Data: Initial multiplier $\boldsymbol{\lambda}^{(0)} \in \mathrm{L}^\infty$, step $\rho > 0$;
Result: Optimal solution $\mathbf{U}^\sharp$ and multiplier $\boldsymbol{\lambda}^\sharp$;
repeat

$$\mathbf{U}^{(k)} \in \arg\min_{\mathbf{U} \in \mathcal{U}^{\mathrm{ad}}} \Big\{ J(\mathbf{U}) + \langle \boldsymbol{\lambda}^{(k)}, \Theta(\mathbf{U}) \rangle \Big\} ,$$

$$\boldsymbol{\lambda}^{(k+1)} = \boldsymbol{\lambda}^{(k)} + \rho\, \Theta\big(\mathbf{U}^{(k)}\big) .$$

until $\Theta(\mathbf{U}^{(k)}) = 0$;

Remark: numerically, other update rules (e.g. quasi-Newton) can be used, convergence being proven when we find a multiplier $\boldsymbol{\lambda}^{(k)}$ such that $\Theta\big(\mathbf{U}^{(k)}\big) = 0$.

Existence of Solution

Theorem

Assume that:

1. the constraint set $\mathcal{U}^{\mathrm{ad}}$ is weakly* closed;
2. $\Theta : \mathcal{U} \to \mathcal{V}$ is affine, weakly* continuous;
3. the objective function $J : \mathcal{U} \to \bar{\mathbb{R}}$ is weak* lower semicontinuous and coercive on $\mathcal{U}^{\mathrm{ad}}$;
4. there exists an admissible control.

Then the primal problem admits at least one solution. Moreover for any $\boldsymbol{\lambda} \in \mathrm{L}^\infty\big(\Omega, \mathcal{F}, \mathbb{P}; \mathbb{R}^m\big)$

$$\operatorname*{arg\,min}_{\mathbf{U} \in \mathcal{U}^{\mathrm{ad}}} \Big\{ J(\mathbf{U}) + \langle \boldsymbol{\lambda} \,, \Theta(\mathbf{U}) \rangle \Big\} \neq \emptyset \; .$$

Convergence Result

Theorem

Assume that:

1. $J : \mathcal{U} \to \bar{\mathbb{R}}$ is a proper, weak* lower semicontinuous, Gâteaux-differentiable, a-convex function;
2. $\Theta : \mathcal{U} \to \mathcal{V}$ is affine, weak* continuous and κ-Lipschitz;
3. there exists an admissible control;
4. $\mathcal{U}^{\mathrm{ad}}$ is weak* closed convex;
5. there is an optimal L^1-multiplier to the constraint $\Theta\big(\mathbf{U}\big) = 0$;
6. the step ρ is such that $0 < \rho < \frac{2a}{\kappa}$.

Then, Uzawa algorithm is well defined and there exists a subsequence $\big(\mathbf{U}^{(n_k)}\big)_{k\in\mathbb{N}}$ converging in L^∞ toward the optimal solution $\mathbf{U}^\sharp$ of the primal problem.

Convergence Result

Theorem

Assume that:

1. $J : \mathcal{U} \to \bar{\mathbb{R}}$ is a proper, weak* lower semicontinuous, Gâteaux-differentiable, a-convex function;
2. $\Theta : \mathcal{U} \to \mathcal{V}$ is affine, weak* continuous and κ-Lipschitz;
3. there exists an admissible control;
4. $\mathcal{U}^{\mathrm{ad}}$ is weak* closed convex;
5. there is an optimal L^1-multiplier to the constraint $\Theta(\mathbf{U}) = 0$;
6. the step ρ is such that $0 < \rho < \frac{2a}{\kappa}$.

Then, Uzawa algorithm is well defined and there exists a subsequence $\big(\mathbf{U}^{(n_k)}\big)_{k\in\mathbb{N}}$ converging in L^∞ toward the optimal solution $\mathbf{U}^\sharp$ of the primal problem.

Convergence Result

Theorem

Assume that:

1. $J : \mathcal{U} \to \bar{\mathbb{R}}$ is a proper, weak* lower semicontinuous, Gâteaux-differentiable, a-convex function;
2. $\Theta : \mathcal{U} \to \mathcal{V}$ is affine, weak* continuous and κ-Lipschitz;
3. there exists an admissible control;
4. $\mathcal{U}^{\mathrm{ad}}$ is weak* closed convex;
5. there is an optimal L^1-multiplier to the constraint $\Theta\big(\mathbf{U}\big) = 0$;
6. the step ρ is such that $0 < \rho < \frac{2a}{\kappa}$.

Then, Uzawa algorithm is well defined and there exists a subsequence $\big(\mathbf{U}^{(n_k)}\big)_{k\in\mathbb{N}}$ converging in L^∞ toward the optimal solution $\mathbf{U}^\sharp$ of the primal problem.

Presentation Outline

1. The Idea of Spatial Decomposition
   - Intuition of Spatial Decomposition
   - Stochastic Spatial Decomposition
2. Uzawa Algorithm
   - Presentation of Uzawa Algorithm
   - Uzawa Algorithm in L^∞
   - Existence of Multiplier
3. Dual Approximate Dynamic Programming
   - Presentation of DADP
   - Practical Questions
   - Numerical Results

Standard duality in L^2 spaces

Assume that $\mathcal{U} = \mathrm{L}^2(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^n)$ and $\mathcal{V} = \mathrm{L}^2(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^m)$.

The standard sufficient constraint qualification condition

$$0 \in \mathrm{ri}\Big(\Theta\big(\mathcal{U}^{\mathrm{ad}} \cap \mathrm{dom}(J)\big) + C\Big) ,$$

is scarcely satisfied in such a stochastic setting.

Proposition

If the σ-algebra $\mathcal{A}$ is not finite modulo $\mathbb{P}$, then for any subset $U^{\mathrm{ad}} \subset \mathbb{R}^n$ that is not an affine subspace, the set

$$\mathcal{U}^{\mathrm{ad}} = \Big\{ \mathbf{U} \in \mathrm{L}^p(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^n) \mid \mathbf{U} \in U^{\mathrm{ad}} \quad \mathbb{P}-a.s. \Big\} ,$$

has an empty relative interior in L^p, for any $p < +\infty$.

Standard duality in L^2 spaces

Assume that $\mathcal{U} = \mathrm{L}^2\big(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^n\big)$ and $\mathcal{V} = \mathrm{L}^2\big(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^m\big)$.

The standard sufficient constraint qualification condition

$$0 \in \mathrm{ri}\Big(\Theta\big(\mathcal{U}^{\mathrm{ad}} \cap \mathrm{dom}(J)\big) + C\Big) ,$$

is scarcely satisfied in such a stochastic setting.

Proposition

If the σ-algebra $\mathcal{A}$ is not finite modulo $\mathbb{P}$, then for any subset $U^{\mathrm{ad}} \subset \mathbb{R}^n$ that is not an affine subspace, the set

$$\mathcal{U}^{\mathrm{ad}} = \Big\{\mathbf{U} \in \mathrm{L}^p\big(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^n\big) \mid \mathbf{U} \in U^{\mathrm{ad}} \quad \mathbb{P} - a.s.\Big\} ,$$

has an empty relative interior in L^p, for any $p < +\infty$.

Standard duality in L^2 spaces II

Consider the following optimization problem:

$$\begin{aligned} \inf_{u_0, \mathbf{U}_1} \quad & u_0^2 + \mathbb{E}\big[(\mathbf{U}_1 + \alpha)^2\big] \ , \\ \text{s.t.} \quad & u_0 \geq a \ , \\ & \mathbf{U}_1 \geq 0 \ , \\ & u_0 - \mathbf{U}_1 \geq \mathbf{W} \ , \qquad \text{to be dualized} \end{aligned}$$

where $\mathbf{W}$ is a random variable uniform on $[1, 2]$.

For $a < 2$:

- we can construct a maximizing sequence of multipliers for the dual problem that does not converge in L^2;
- this is a case of *non relatively complete recourse* (constraints on $\mathbf{U}_1$ induce stronger constraint on u_0;
- however there exists an optimal multiplier in $(\mathrm{L}^\infty)^\star$

Standard duality in L^2 spaces II

Consider the following optimization problem:

$$\begin{aligned} \inf_{u_0,\mathbf{U}_1} \quad & u_0^2 + \mathbb{E}\left[(\mathbf{U}_1 + \alpha)^2\right] , \\ \text{s.t.} \quad & u_0 \geq a , \\ & \mathbf{U}_1 \geq 0 , \\ & u_0 - \mathbf{U}_1 \geq \mathbf{W} , \qquad \text{to be dualized} \end{aligned}$$

where $\mathbf{W}$ is a random variable uniform on $[1, 2]$.

For $a < 2$:

- we can construct a maximizing sequence of multipliers for the dual problem that does not converge in L^2;
- this is a case of *non relatively complete recourse* (constraints on $\mathbf{U}_1$ induce stronger constraint on u_0;
- however there exists an optimal multiplier in $\left(\mathrm{L}^\infty\right)^\star$

Constraint qualification in $\left(\mathrm{L}^\infty, \mathrm{L}^1\right)$

From now on, we assume that

$$\begin{aligned}
\mathcal{U} &= \mathrm{L}^\infty\left(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^n\right) , \\
\mathcal{V} &= \mathrm{L}^\infty\left(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^m\right) , \\
C &= \{0\} ,
\end{aligned}$$

where the σ-algebra $\mathcal{A}$ is not finite modulo $\mathbb{P}$.[1]

We consider the pairing $\left(\mathrm{L}^\infty, \mathrm{L}^1\right)$ with the following topologies:

- $\sigma\left(\mathrm{L}^\infty, \mathrm{L}^1\right)$: weak* topology on L^∞ (coarsest topology such that all the L^1-linear forms are continuous),
- $\tau\left(\mathrm{L}^\infty, \mathrm{L}^1\right)$: Mackey-topology on L^∞ (finest topology such that the continuous linear forms are only the L^1-linear forms).

[1] If the σ-algebra is finite modulo $\mathbb{P}$, $\mathcal{U}$ and $\mathcal{V}$ are finite dimensional spaces.

Weak* closedness of linear subspaces of L^∞

Proposition

Let $\Theta : \mathrm{L}^\infty\big(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^n\big) \to \mathrm{L}^\infty\big(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^m\big)$ *be a linear operator, and assume that there exists a linear operator* $\Theta^\dagger : \mathrm{L}^1\big(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^m\big) \to \mathrm{L}^1\big(\Omega, \mathcal{A}, \mathbb{P}; \mathbb{R}^n\big)$ *such that:*

$$\big\langle \mathbf{V} \, , \Theta(\mathbf{U}) \big\rangle = \big\langle \Theta^\dagger(\mathbf{V}) \, , \mathbf{U} \big\rangle \quad \forall \mathbf{U}, \; \forall \mathbf{V} \; .$$

Then the linear operator Θ *is weak** *continuous.*

Applications

- $\Theta(\mathbf{U}) = \mathbf{U} - \mathbb{E}\big[\mathbf{U} \bigm| \mathcal{B}\big]$: non-anticipativity constraints,
- $\Theta(\mathbf{U}) = A\mathbf{U}$ with $A \in \mathcal{M}_{m,n}(\mathbb{R})$: finite number of constraints.

A duality theorem

$$(\mathcal{P}) \qquad \min_{\mathbf{U}\in\mathcal{U}} J(\mathbf{U}) \quad \text{s.t.} \quad \Theta(\mathbf{U}) = 0 \;,$$

with $J(\mathbf{U}) = \mathbb{E}\big[j(\mathbf{U},\mathbf{W})\big]$.

Theorem

Assume that j is a convex normal integrand, that J is continuous in the Mackey topology at some point $\mathbf{U}_0$ such that $\Theta(\mathbf{U}_0) = 0$, and that Θ is weak* continuous on $\mathrm{L}^\infty\big(\Omega,\mathcal{A},\mathbb{P};\mathbb{R}^n\big)$.
Then, $\mathbf{U}^\sharp \in \mathcal{U}$ is an optimal solution of Problem $\big(\mathcal{P}\big)$ if and only if there exists $\boldsymbol{\lambda}^\sharp \in \mathrm{L}^1\big(\Omega,\mathcal{A},\mathbb{P};\mathbb{R}^m\big)$ such that

- $\mathbf{U}^\sharp \in \arg\min_{\mathbf{U}\in\mathcal{U}} \mathbb{E}\Big[j(\mathbf{U},\mathbf{W}) + \boldsymbol{\lambda}^\sharp\cdot\Theta(\mathbf{U})\Big]$,
- $\Theta(\mathbf{U}^\sharp) = 0$.

Extension of a result given by R. Wets for non-anticipativity constraints.

Presentation Outline

Dual approximation as constraint relaxation

The original problem is (abstract form)

$$\min_{\mathbf{U}\in\mathcal{U}} \quad J(\mathbf{U})$$
$$s.t. \quad \Theta(\mathbf{U}) = 0$$

written as

$$\min_{\mathbf{U}\in\mathcal{U}} \max_{\boldsymbol{\lambda}} \quad J(\mathbf{U}) + \mathbb{E}\big[\langle \boldsymbol{\lambda}, \Theta(\mathbf{U})\rangle\big]$$

Subsituting $\boldsymbol{\lambda}$ by $\mathbb{E}\big(\boldsymbol{\lambda}\big|\mathbf{Y}\big)$ gives

$$\min_{\mathbf{U}\in\mathcal{U}} \max_{\boldsymbol{\lambda}} \quad J(\mathbf{U}) + \mathbb{E}\Big[\big\langle \mathbb{E}\big(\boldsymbol{\lambda}\big|\mathbf{Y}\big), \Theta(\mathbf{U})\big\rangle\Big]$$

Dual approximation as constraint relaxation

The original problem is (abstract form)

$$\min_{\mathbf{U}\in\mathcal{U}} \quad J(\mathbf{U})$$
$$s.t. \quad \Theta(\mathbf{U}) = 0$$

written as

$$\min_{\mathbf{U}\in\mathcal{U}} \max_{\boldsymbol{\lambda}} \quad J(\mathbf{U}) + \mathbb{E}\big[\langle \boldsymbol{\lambda}, \Theta(\mathbf{U})\rangle\big]$$

Subsituting $\boldsymbol{\lambda}$ by $\mathbb{E}\big(\boldsymbol{\lambda}\big|\mathbf{Y}\big)$ gives

$$\min_{\mathbf{U}\in\mathcal{U}} \max_{\boldsymbol{\lambda}} \quad J(\mathbf{U}) + \mathbb{E}\Big[\big\langle \boldsymbol{\lambda}, \mathbb{E}\big(\Theta(\mathbf{U})\big|\mathbf{Y}\big)\big\rangle\Big]$$

Dual approximation as constraint relaxation

The original problem is (abstract form)

$$\min_{\mathbf{U}\in\mathcal{U}} \quad J(\mathbf{U})$$
$$s.t. \quad \Theta(\mathbf{U}) = 0$$

written as

$$\min_{\mathbf{U}\in\mathcal{U}} \max_{\boldsymbol{\lambda}} \quad J(\mathbf{U}) + \mathbb{E}\big[\langle \boldsymbol{\lambda}, \Theta(\mathbf{U})\rangle\big]$$

Subsituting $\boldsymbol{\lambda}$ by $\mathbb{E}\big(\boldsymbol{\lambda}\big|\mathbf{Y}\big)$ gives

$$\min_{\mathbf{U}\in\mathcal{U}} \max_{\boldsymbol{\lambda}} \quad J(\mathbf{U}) + \mathbb{E}\Big[\big\langle \boldsymbol{\lambda}, \mathbb{E}\big(\Theta(\mathbf{U})\big|\mathbf{Y}\big)\big\rangle\Big]$$

equivalent to

$$\min_{\mathbf{U}\in\mathcal{U}} \quad J(\mathbf{U})$$
$$s.t. \quad \mathbb{E}\big(\Theta(\mathbf{U})\big|\mathbf{Y}\big) = 0$$

Recall of the Multistage Problem

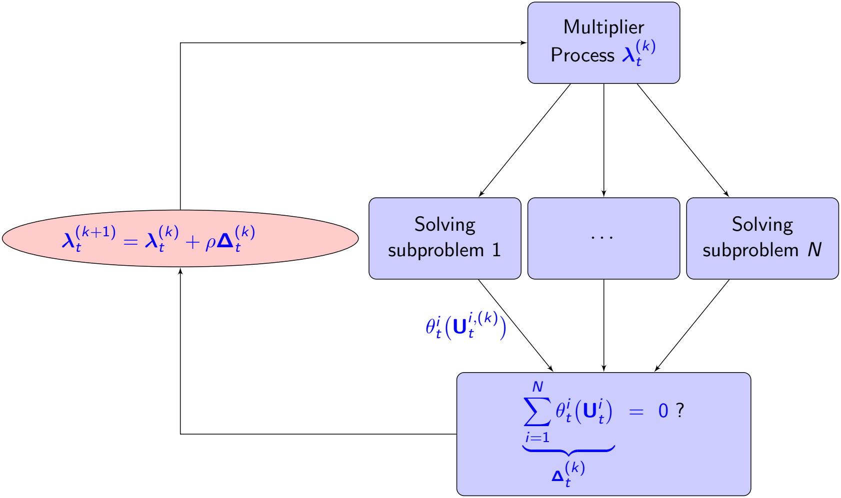
$$\min_{\mathbf{U}} \quad \sum_{i=1}^{N} \mathbb{E}\Big[\sum_{t=1}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T) \Big]$$

$$\mathbf{X}_{t+1}^i = f_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}), \qquad \mathbf{X}_0^i = x_0^i$$

$$\mathbf{U}_t^i \in \mathcal{U}_{t,i}^{ad}, \qquad \mathbf{U}_t^i \preceq \mathcal{F}_t$$

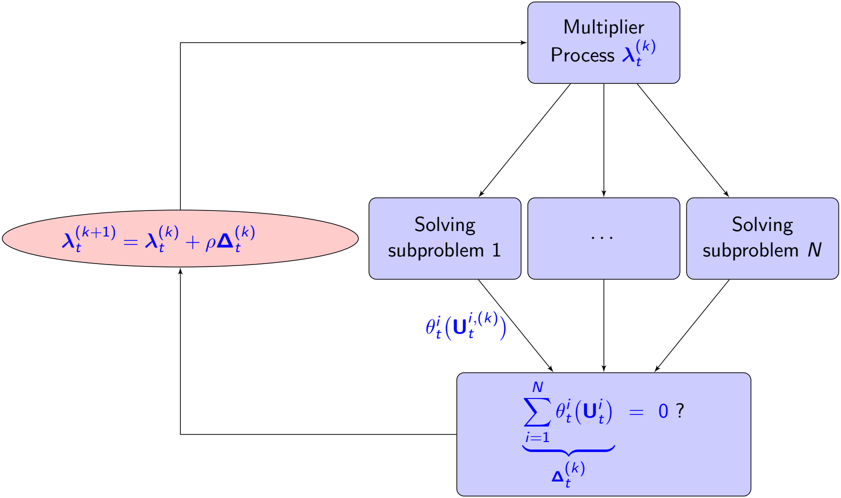
$$\sum_{i=1}^{N} \theta_t^i(\mathbf{U}_t^i) = 0 \qquad \rightsquigarrow \boldsymbol{\lambda}_t$$

Stochastic spatial decomposition scheme

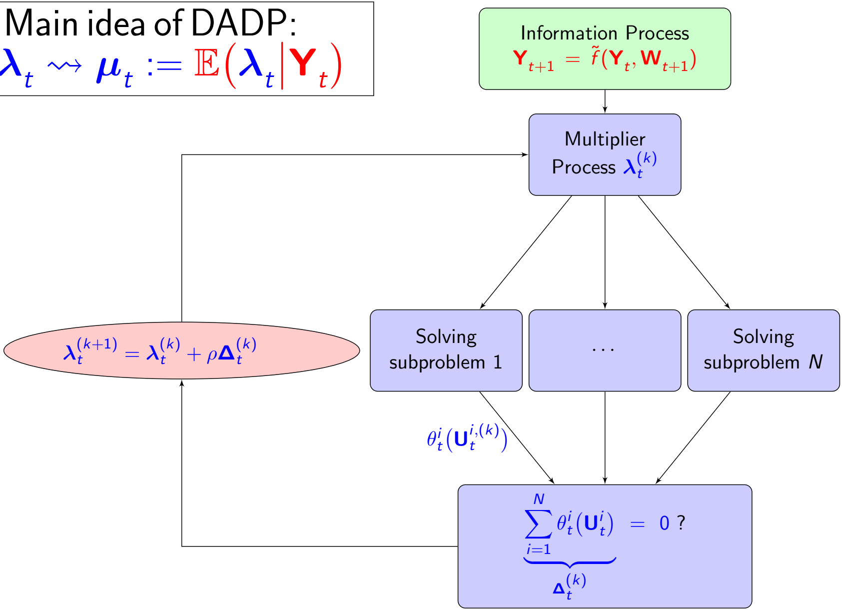


Main idea of DADP:

$$\boldsymbol{\lambda}_t \rightsquigarrow \boldsymbol{\mu}_t := \mathbb{E}\big(\boldsymbol{\lambda}_t \big| \mathbf{Y}_t\big)$$

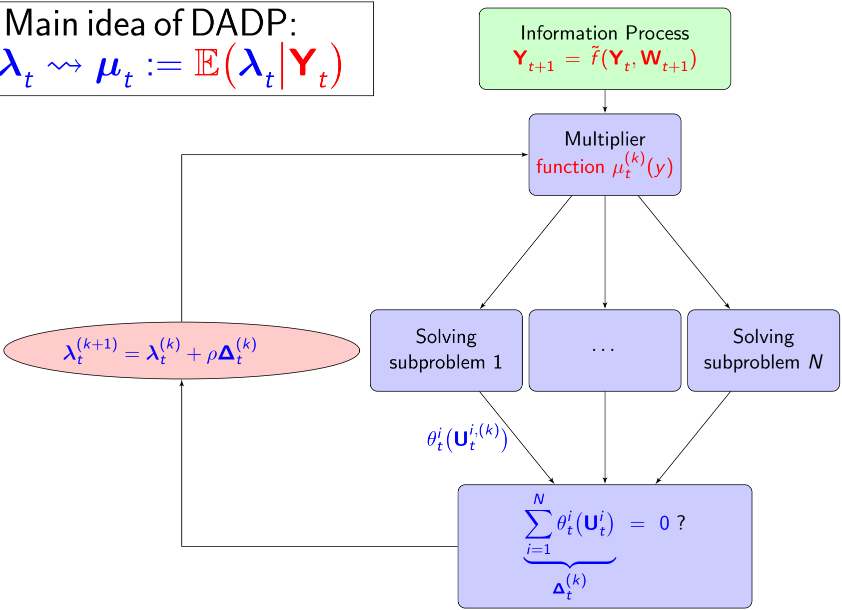


Main idea of DADP:
$\boldsymbol{\lambda}_t \rightsquigarrow \boldsymbol{\mu}_t := \mathbb{E}\big(\boldsymbol{\lambda}_t \big| \mathbf{Y}_t\big)$

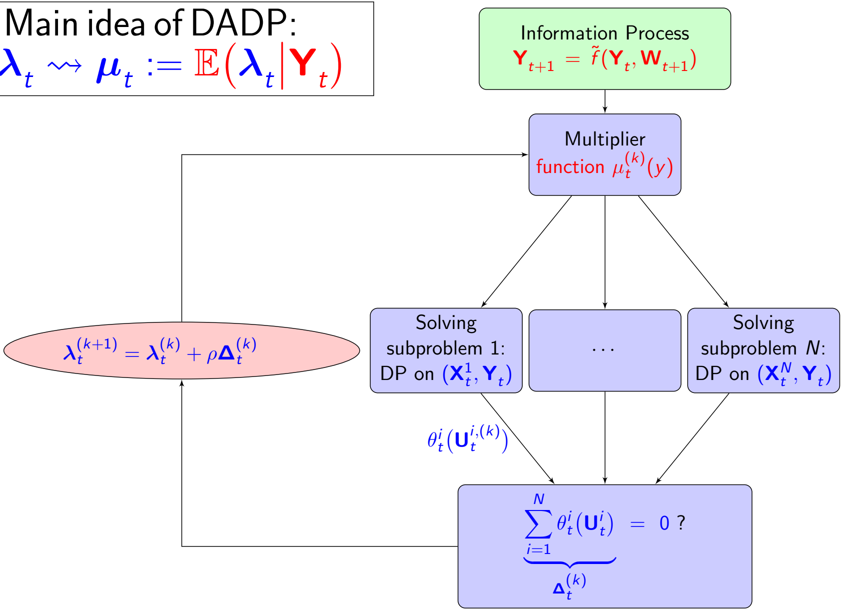


Main idea of DADP:

$$\boldsymbol{\lambda}_t \rightsquigarrow \boldsymbol{\mu}_t := \mathbb{E}\big(\boldsymbol{\lambda}_t \big| \mathbf{Y}_t\big)$$

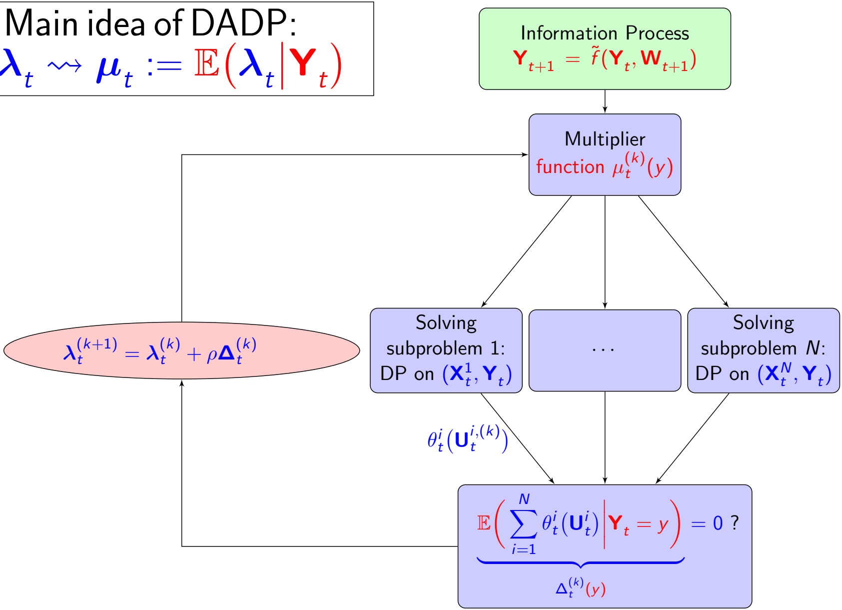


Main idea of DADP:
$\boldsymbol{\lambda}_t \rightsquigarrow \boldsymbol{\mu}_t := \mathbb{E}\big(\boldsymbol{\lambda}_t \big| \mathbf{Y}_t\big)$



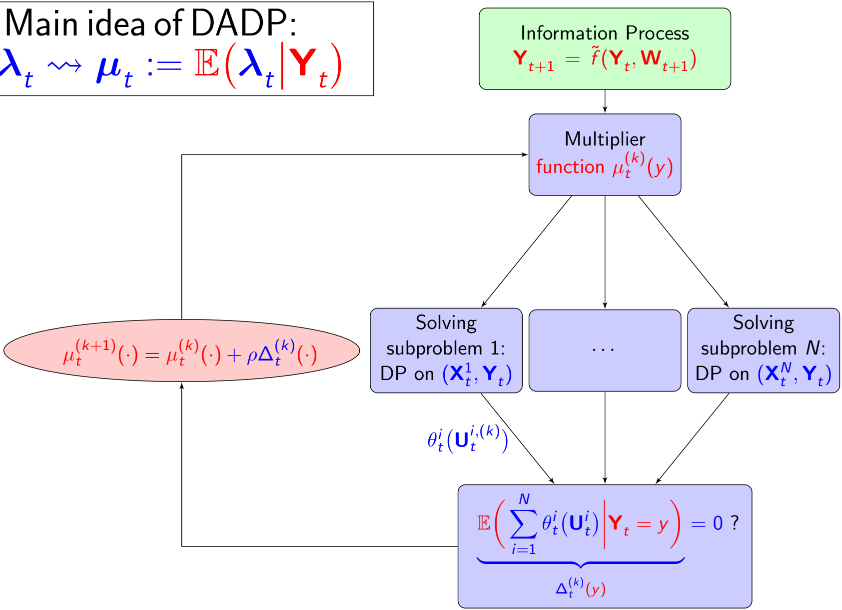
Main idea of DADP:

$$\boldsymbol{\lambda}_t \rightsquigarrow \boldsymbol{\mu}_t := \mathbb{E}\big(\boldsymbol{\lambda}_t \big| \mathbf{Y}_t\big)$$



Main idea of DADP:

$$\boldsymbol{\lambda}_t \rightsquigarrow \boldsymbol{\mu}_t := \mathbb{E}\big(\boldsymbol{\lambda}_t \big| \mathbf{Y}_t\big)$$



Global problem:

$$\min_{\{\mathbf{U}_t^i\}_{i,t}} \quad \sum_{i=1}^{N} \mathbb{E}\Big[\sum_{t=1}^{T} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K(\mathbf{X}_T^i)\Big]$$

$$\mathbf{X}_{t+1}^i = f_t(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}), \qquad \mathbf{X}_0^i = x_0^i$$

$$\mathbf{U}_t^i \in \mathcal{U}_{t,i}^{ad}, \qquad \mathbf{U}_t^i \preceq \mathcal{F}_t$$

$$\sum_{i=1}^{n} \theta_t^i(\mathbf{U}_t^i) = 0$$

Solved by DP with state $(\mathbf{X}_t^1, \dots, \mathbf{X}_t^N)$:

$$V_t(x) = \min_{\{\mathbf{U}_t^i\}_i} \quad \sum_{i=1}^{N} \mathbb{E}\Big[L_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}) + V_{t+1}(\mathbf{X}_{t+1})\Big]$$

$$\mathbf{X}_{t+1}^i = f_t(x_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}),$$

$$u_t^i \in \mathcal{U}_{t,i}^{ad},$$

$$\sum_{i=1}^{n} \theta_t^i(\mathbf{U}_t^i) = 0$$

Subproblem of Stochastic Decomposition

$$\min_{\{\mathbf{U}_t^i\}_t} \quad \mathbb{E}\Big[\sum_{t=1}^{T} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + \langle \boldsymbol{\lambda}_t, \theta_t(\mathbf{U}_t^i)\rangle + K(\mathbf{X}_T^i)\Big]$$

$$\mathbf{X}_{t+1}^i = f_t(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}), \qquad \mathbf{X}_0^i = x_0^i$$

$$\mathbf{U}_t^i \in \mathcal{U}_{t,i}^{ad}, \qquad \mathbf{U}_t^i \preceq \mathcal{F}_t$$

Solved by DP with state $(\mathbf{W}_1, \dots, \mathbf{W}_t)$:

$$V_t\big(\{w_\tau\}_1^{t-1}\big) = \min_{\{\mathbf{U}_t^i\}} \quad \mathbb{E}\Big[L_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}) + \langle \boldsymbol{\lambda}_t, \theta_t(\mathbf{U}_t^i)\rangle + V_{t+1}\big(\{\mathbf{W}_\tau\}_1^t\big) \Big| \{\mathbf{W}_\tau\}_1^{t-1} = \{w_\tau\}_1^{t-1}\Big]$$

$$\mathbf{X}_{t+1}^i = f_t(x_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}),$$

$$u_t^i \in \mathcal{U}_{t,i}^{ad},$$

Subproblem of DADP

$$\min_{\{\mathbf{U}_t^i\}_t} \quad \mathbb{E}\Big[\sum_{t=1}^{T} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + \langle \mu_t(\mathbf{Y}_t), \theta_t(\mathbf{U}_t^i)\rangle + K(\mathbf{X}_T^i)\Big]$$

$$\mathbf{X}_{t+1}^i = f_t(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}), \qquad \mathbf{X}_0^i = x_0^i$$

$$\mathbf{U}_t^i \in \mathcal{U}_{t,i}^{ad}, \qquad \mathbf{U}_t^i \preceq \mathcal{F}_t$$

$$\mathbf{Y}_{t+1} = \tilde{f}_t(\mathbf{Y}_t, \mathbf{W}_{t+1})$$

Solved by DP with state $(\mathbf{X}_t^i, \mathbf{Y}_t)$:

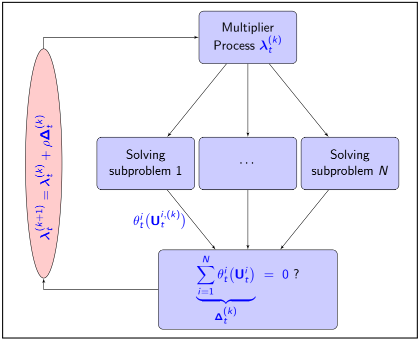
$$V_t^i(x, y) = \min_{\{\mathbf{U}_t^i\}} \quad \mathbb{E}\Big[L_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}) + \langle \mu_t(\mathbf{Y}_t), \theta_t(\mathbf{U}_t^i)\rangle + V_{t+1}(\mathbf{X}_{t+1}, \mathbf{Y}_{t+1})\Big]$$

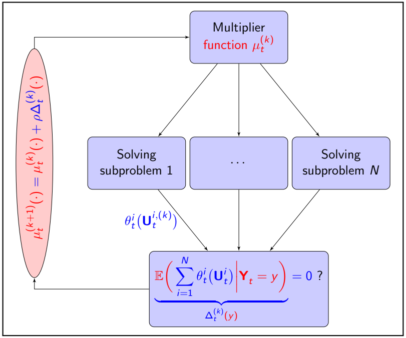
$$\mathbf{X}_{t+1}^i = f_t(x_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}),$$

$$u_t^i \in \mathcal{U}_{t,i}^{ad},$$

$$\mathbf{Y}_{t+1} = \tilde{f}_t(y, \mathbf{W}_{t+1})$$

Main idea of DADP: $\boldsymbol{\lambda}_t \rightsquigarrow \boldsymbol{\mu}_t := \mathbb{E}\big(\boldsymbol{\lambda}_t \big| \mathbf{Y}_t\big)$





Main problems:

- Subproblems not easily solvable by DP
- $\boldsymbol{\lambda}^{(k)}$ live in a huge space

Advantages:

- Subproblems solvable by DP with state $(\mathbf{X}_t^i, \mathbf{Y}_t)$
- $\mu^{(k)}$ live in a smaller space

3 Interpretations of DADP

- DADP as an approximation of the optimal multiplier

$$\boldsymbol{\lambda}_t \quad \rightsquigarrow \quad \mathbb{E}\big(\boldsymbol{\lambda}_t \big| \mathbf{Y}_t\big) \ .$$

- DADP as a decision-rule approach in the dual

$$\max_{\boldsymbol{\lambda}} \min_{\mathbf{U}} L\big(\boldsymbol{\lambda}, \mathbf{U}\big) \quad \rightsquigarrow \quad \max_{\boldsymbol{\lambda}_t \preceq \mathbf{Y}_t} \min_{\mathbf{U}} L\big(\boldsymbol{\lambda}, \mathbf{U}\big) \ .$$

- DADP as a constraint relaxation

$$\sum_{i=1}^{n} \theta_t^i\big(\mathbf{U}_t^i\big) = 0 \quad \rightsquigarrow \quad \mathbb{E}\Big(\sum_{i=1}^{n} \theta_t^i\big(\mathbf{U}_t^i\big) \Big| \mathbf{Y}_t\Big) = 0 \ .$$

Theoretical Results

- Consistence of the approximation (if we consider a sequence of approximated problems).
- Existence of multiplier of the coupling constraint.
- Convergence of the decomposition algorithm for a given relaxation.
- Lower and upper bounds on the original problem.
- A posteriori verification allowing for better multiplier update.

Presentation Outline

Choosing an Information Process $\mathbf{Y}$

- Perfect memory: $\mathbf{Y}_t^i = \big(\mathbf{W}_0, \dots, \mathbf{W}_t\big)$.
  $\leadsto$ equivalent to original problem, no numerical gain.
- Minimal information: $\mathbf{Y}_t^i \equiv \mathrm{cste}$.
  $\leadsto$ equivalent to replacing a.s. constraint by expected constraint. Subproblems solved efficiently (state $\mathbf{X}_t^i$), multiplier is deterministic.
- Static information: $\mathbf{Y}_t^i = h_t^i\big(\mathbf{W}_t\big)$.
  $\leadsto$ Subproblems solved efficiently (state $\mathbf{X}_t^i$).
- Dynamic information: $\mathbf{Y}_{t+1}^i = h_t^i\big(\mathbf{Y}_t^i, \mathbf{W}_{t+1}\big)$.
  $\leadsto$ A number of possibilities. Some ideas:
  - mimicking the trajectory of the state of another unit (phantom state),
  - mimicking the control of other units,
  - Markov chain representing rougly the general state of the system.

Numerical Advantages of a finitely supported $\mathbf{Y}$

- Assume that each noise $\mathbf{W}_t$ take w values, and the constraint function take value in $\mathbb{R}$.
- Then the multiplier $\boldsymbol{\lambda}_t$ of the almost sure constraint at time t lives in $\mathbb{R}^{wt}$.
- Assume that the information process at time t take y values, then the multiplier of the relaxed constraint $\boldsymbol{\mu}_t$ lives in $\mathbb{R}^{y}$.
- Moreover each subproblems take “only” roughly y times more computational effort to solve than the subproblem with local state $\mathbf{X}_t^i$.

Back to Admissibility

- Consider an information process $\mathbf{Y}_{t+1} = \tilde{f}_t(\mathbf{Y}_t, \mathbf{W}_{t+1})$.
- For a multiplier process $\mu_t^{(k)}$ we obtain local Bellman function

$$\tilde{V}_t^i(x^i, y) = \min_{u^i} \quad \mathbb{E}\Big[L_t^i\big(x^i, u^i, \mathbf{W}_{t+1}\big) + \tilde{V}_t^i\big(x_{t+1}^i, y_{t+1}\big)\Big]$$
$$\mathbf{X}_{t+1}^i = f_t\big(x^i, u^i, \mathbf{W}_{t+1}\big)$$
$$\mathbf{Y}_{t+1} = \tilde{f}_t\big(y, \mathbf{W}_{t+1}\big)$$

- An admissible strategy is given by

$$\pi_t^{\mathrm{ad}}(x, y) \in \mathop{\arg\min}_{\{u^i\}_{i \in [1,N]}} \quad \mathbb{E}\Big[\sum_{i=1}^{N} \Big(L_t^i\big(x^i, u^i, \mathbf{W}_{t+1}\big) + \tilde{V}_t^i\big(\mathbf{X}_{t+1}, \mathbf{Y}_{t+1}\big)\Big)\Big]$$
$$\mathbf{X}_{t+1}^i = f_t^i\big(x^i, u^i, \mathbf{W}_{t+1}\big), \qquad \forall i$$
$$\mathbf{Y}_{t+1} = \tilde{f}_t\big(y, \mathbf{W}_{t+1}\big)$$

Presentation Outline

Problem Specification

- We consider a 3 dam problem, over 12 time steps.
- We relax each constraint with a given information process $\mathbf{Y}^i$.
- All random variable are discrete (noise, control, state).

- We test the following information processes:

  Constant information equivalent to replace the a.s. constraint by an expected constraint,

  Part of noise the information process is the inflow of the above dam $\mathbf{Y}_t^i = \mathbf{W}_t^{i-1}$,

  Phantom state the information process mimick the optimal trajectory of the state of the first dam (by statistical regression over the known optimal trajectory in this case)

Numerical Results on the 3 Dams Example

	DADP - $\mathbb{E}$	DADP - $\mathbf{W}^{i-1}$	DADP - dyn.	DP
Nb of it.	165	170	25	1
Time (min)	2	3	67	41
Lower Bound	-1.386×10^6	-1.379×10^6	-1.373×10^6	
Final Value	-1.335×10^6	-1.321×10^6	-1.344×10^6	-1.366×10^6
Loss	-2.3%	-3.3%	-1.6%	ref.

Table: Numerical results on the 3-dam problem

Summing up DADP

- Choose an information process $\mathbf{Y}$ following $\mathbf{Y}_{t+1} = \tilde{f}_t(\mathbf{Y}_t, \mathbf{W}_{t+1})$.
- We relax the almost sure coupling constraint into a conditional expectation one and apply a price decomposition scheme to the relaxed problem.
- The subproblems can be solved by dynamic programming with the state $(\mathbf{X}^i_t, \mathbf{Y}_t)$.
- We give a consistency result (family of information process), a convergence result (fixed information process) and an existence of multiplier condition.

The end

Thanks for your attention!

More information[2] on theoretical results tomorrow at ENPC, amphi Caquot I, (14h).

[2] and hopefully some champagne