Joint production and energy supply planning of an industrial microgrid

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Abstract

We consider the problem of jointly optimizing the daily production planning and energy supply management of an industrial complex, with manufacturing processes, renewable energies and energy storage systems. It is naturally formulated as a mixed-integer multistage stochastic problem. This problem is challenging for three main reasons: there is a large number of time steps (typically 24), renewable energies are uncertain and uncontrollable, and we need binary variables modeling hard constraints. We discuss various solution strategies, in particular Model Predictive Control, Dynamic Programming, and heuristics based on the Stochastic Dual Dynamic Programming algorithm. We compare these strategies on two variants of the problem: with or without day-ahead energy purchases.

1 Introduction

The latest Intergovernmental Panel on Climate Change (IPCC) warns us yet again about the consequences of climate change and incites governments, industries and citizens to change accordingly. The COP27, held in November 2022, set up a clear objective of securing global net-zero emissions by mid-century. Therefore, the industry, counting for one-fourth of global emissions (6th IPCC report), must take strong actions to reduce them. In this respect, the Clean Energy Ministerial Industrial Deep Decarbonisation Initiative (IDDI) calls for a change in the energy supply, as industries consume fuel massively to produce local energy, especially steel and cement production. To put things in perspective, renewable generation represented only 16.9% of electricity generation in the industrial sector in 2020, which is far less than its share in global electricity generation, up to 28% in 2020, according to the International Energy Agency (IEA), see their Tracking Industry 2021 report [Intc] and their Global Energy Review 2021 report [Inta]. For instance, microgrids are an alternative energy supply model. They are defined (see *e.g.,* [HPG18]) as a small-scale power grid that can operate independently or collaboratively with the power grid. Generally, they are made of Energy Storage Systems (ESS), renewable energy generation units (wind turbines, solar panels) and consumption units (factories, buildings, etc.). With recent technological advances, such energy systems are becoming more efficient and cheaper to install and operate. Moreover, some governments subsidize energy transition efforts, which encourages factories to invest in onsite renewable energy. For instance, The Fairfield, California brewery[1] has invested in a solar array and wind turbine which provide an average of 30% of its electricity needs. Another example is the French company E.Leclerc which equips some of its hypermarkets with solar generation.

In a recent review of energy sustainability in manufacturing systems [RM21], the authors point out that, in most papers, the problem of energy management is decoupled from manufacturing operations. However, they argue that this decoupling is not realistic as the two problems are interdependent, and suggest that research should be conducted on solving those problems jointly. In this paper, we address this issue by proposing a joint production and energy supply planning problem.

Unfortunately, incorporating renewable energies in the supply mix is challenging as they are intermittent, unpredictable and uncontrollable. To counteract these defects it is often suggested to add an ESS (we refer to [Geo+21] for an overview of the available ESS). Doing so allows transferring energy across time steps, making it controllable and compensating for intermittency. Unpredictability of the renewable production requires going from a deterministic formulation to a stochastic formulation. Indeed, a classical deterministic problem is often misleading and optimistic about the potential of the ESS. Unfortunately, multistage stochastic problems are known to be numerically challenging (see *e.g.,* [Sha06]). Starting from a standard scheduling industrial problem, we consider relying on an onsite microgrid to provide an alternative energy supply to the main grid. We obtain a mixed-integer multistage stochastic problem optimizing jointly the production planning and the energy supply management of an industrial facility with in advance and intraday energy purchases.

1.1 The industrial microgrid management problem

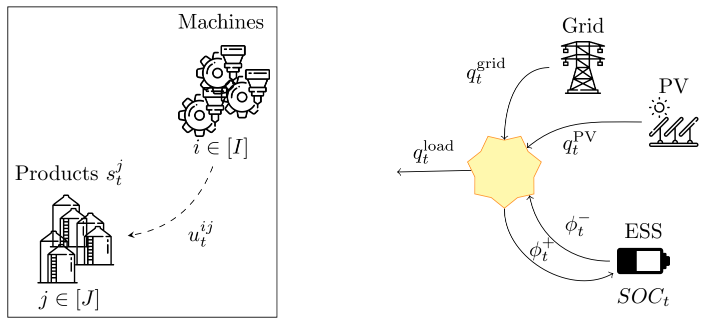
Building renewable energy production and storage management systems to supply an industrial facility is a complex task. One of the questions at hand is the financial rentability of such a system, which is not guaranteed. To incite the industry to invest in renewable energies, we need economic guarantees: see [Intb] and [Intd] for an overview of clean energy transition costs in 2021. The economic viability of a microgrid is based on controlling the investment costs and managing the microgrid efficiently. In this paper, we do not discuss the investment part but focus on the operational part.

We consider a facility with I machines that produce up to J types of products that can be stored (see fig. 1a). Our goal is to provide the facility with a joint production and energy supply planning, on a discrete horizon $t \in [T]$. The planning should minimize the total expected cost (economic, environmental and labor) while satisfying production targets and technical constraints.

Depending on the facility at hand, many technical constraints need to be satisfied. We can classify them into three types. First, *physical constraints* are induced by the machines at hand. For example, most machines, such as grinders or plastic extruders, require warming up before being operational. Another straightforward example comes from the food industry, where machines need to be cleaned up to reconfigure the production line. Second, *process constraints* which correspond to precedence constraints mandating sequential execution of some tasks (usually called flow-shop problems). For instance, in a chocolate factory, every batch production will follow in order: cleaning, roasting, shell removing, grinding and conching. Finally, *implied constraints* model decision-maker preferences or human resources constraints. For example, the decision maker may limit the number of re-starts to limit wear-off, if a machine is hard to access or for human power reasons.

Most of the above constraints are modeled with binary variables. Thus, even though we focus here on a specific problem, the developed approach can be transposed to a large variety of problems. In this paper,

[1] `https://www.anheuser-busch.com/breweries/fairfield-ca`



(a) Factory Structure

(b) Energy exchanges

Figure 1: Industrial Management Problem

we consider a problem with bounded production and set-up costs. In addition, we consider shared resource constraints such that some products cannot be produced simultaneously. Factory energy needs, proportional to production, are met with electricity from a main grid or produced onsite by a micro-grid consisting of solar panels coupled with an energy system storage (ESS) see fig. 1b.

Electricity from the main grid can be purchased through two different contracts, usually cumulated: Intra-day contract where prices are fixed annually, the factory pays the energy extracted from the main grid at a given time t; In-advance contract where the factory buys energy blocks in advance (*e.g.,* a day ahead of production) at a preferential rate. Decisions are made adjusting energy purchases based on intra-day rates in real-time.

1.2 Literature review

We consider a problem coupling production planning and energy supply management. Taken separately, each problem has been widely studied, but considering them simultaneously is less common, especially when taking into account uncertainty, leading to large multistage stochastic optimization problem. In this section, we review the state-of-the-art of energy-aware production planning under uncertainties.

A typical angle for energy-aware production systems is to minimize energy waste, see the reviews [Bän+21], [BG16] and references therein. This part of the literature looks for production plans, or scheduling, that are more energy efficient, adapting tools from well-studied problems like single or parallel machine scheduling, job-shop, flow-shop or lot-sizing [2]. However, few papers discuss the economic impact of integrating renewable energy sources on site: indeed, the industrial energy supply is traditionally guaranteed by an external grid. In their survey [Bän+21], Bänsch et al. count 8 articles (out of 192) that consider an onsite energy generation and an ESS. The literature lacks research on industrial problems with distributed generation systems, though, they are widely studied on their own. We refer to the review [Alo+22] where Alonso-Travesset et al. focuses on recent studies on models under uncertainties in distributed generation systems. They highlight the necessity of properly taking into account uncertainties in those problems, in particular regarding renewable energy generation. In the problem considered here, the main source of uncertainty comes from renewable energies. There are two main ways of handling uncertainty: stochastic optimization and robust optimization.

[2] The job-shop problem, see *e.g.,* [Man60], looks for an optimal scheduling plan for n jobs, consisting of operations with precedence constraints, on m machines. The flow-shop problem is a variant of the job-shop problem with a strict order of all operations on all jobs. Finally, a lot-sizing problem optimizes the production quantities of each item at each time step.

1.2.1 Stochastic optimization for operational management

In the first paradigm, we model uncertain variables as random variables with known distribution, usually represented by a scenario tree. Further, as uncertainties are revealed step by step, stochastic problems are often multistage problems that are known to be challenging, while there exist various methods to tackle 2−stage problems *e.g.,* based on Bender's decomposition (see [BL97]). As a result, multistage problems are classically relaxed into 2−stage problems: all decision variables, except the first stage variable, are assumed to be taken with the full knowledge of the uncertainty. This is the strategy adopted by Golari, Fan, and Jin in [GFJ16] to optimize the production planning of interconnected factories each connected to a micro-grid. Biel et al. take this approach as well in [Bie+18] to solve a flow-shop problem under uncertainties regarding wind energy generation. In another article ([WMG20]), Wang, Mason, and Gangammanavar studies a similar problem with multi-objectives (total completion time and energy costs), where selling an energy excess to the main grid is allowed. They propose an ε-constraint algorithm integrated with the L-shaped method ([Bir85]), which is a Benders decomposition adapted to 2−stage stochastic programs. To avoid 2−stage approximations, one can turn to dynamic programming reformulations of the multi-stage stochastic problem. However, vanilla dynamic programming for multistage problems is limited by what is known as the *curse of dimensionality*. In 1991, Pereira and Pinto proposed an efficient algorithm to solve those problems: the Stochastic Dual Dynamic Programming (SDDP) algorithm [PP91]. Since then, SDDP has been widely applied to energy management problems and variants have been derived (see [FR21] for a recent survey). We recall the algorithm and present related literature in section 3.2.

1.2.2 Robust optimization for operational management

In the second paradigm, robust optimization, we consider the worst case in possible uncertainty realizations. This is the choice made by Ruiz Duarte, Fan, and Jin in [RFJ20], where they evaluate the renewable energy integration with an ESS in a factory while optimizing the production planning. This is modeled by a 2−stage problem: in the first stage, a production plan is defined whereas in the second stage, the decisions regarding the energy management system are made to minimize its energy costs under the worst-case energy generation scenario. The robust uncertainty set is determined by statistical tools. Bridging both worlds, Shahandeh, Motamed Nasab, and Li propose in [SML19] to divide random variables into two categories: static and dynamic variables. The idea is to apply robust optimization on one variable category and then stochastic optimization on the other, considering a scenario tree. This results in two hybrid algorithms, mixing robust and stochastic optimization to solve a multistage problem with different uncertainty types.

1.2.3 Price and demand uncertainties

Furthermore, in these industrial problems, the solution is not only affected by renewable energies' variability: costs and demands are other known uncertainty sources. If some articles consider time-of-use (TOU) electricity rates ([Bie+18], [MP13], [Li+17] and [WMG20]), which are fixed prices in contract depending on consumption's times, others consider variable prices. In that respect, Bohlayer et al. ([Boh+20]) and Ierapetritou et al. ([Ier+02]) both study mixed-integer multistage stochastic problems under energy prices uncertainty. See also Fazli Khalaf and Wang ([FW18]) who solve a 2−stage stochastic scheduling problem considering both electricity prices and energy generation as random variables. Finally, in lot-sizing problems, the product demand is often random: Higle and Kempf consider a multistage stochastic program in [HK10] to solve a production planning problem under demand uncertainty, trying to avoid cumulating stocks.

1.2.4 Strategic decisions

We have covered stochastic considerations for operational or tactical production planning problems. We now discuss strategic decisions like investing in renewable energies and ESS, with questions of size, technologies and number of ESS and energy generation units. To adapt their energy mix, factories need to design what distributed generation system is suited for their production. In [FMH21], Fattahi, Mosadegh, and Hasani focus on the planning in mining supply chains with renewable energy investment where at each stage, warehouse or generation systems can be installed. Though economic rentability is crucial, the growing interest in microgrids is driven by environmental concerns. Thus, instead of minimizing energy waste, a more direct approach consists in integrating environmental objectives into costs. For example Li et al., in [Li+17], assess wind and solar generation deployment costs in order to achieve net-zero carbon. On another note, microgrids bring flexibility and energy independence. In [Pha+19], Pham et al. extend Golari, Fan, and Jin's work by considering both stochastic demand and microgrid sizing. Their goal is to determine if it is economically viable to provide the system with only renewable energies.

Investing in microgrids doesn't require only sizing but also investigating the different existing technologies and their characteristics. In [Tsi+21], Tsianikas et al. study the capacity extension problem as well as the different storage technologies. An interesting take on the subject is given in [HBF15]: when most micro-grid investment models consider the ESS sizing at the beginning, Hajipour, Bozorg, and Fotuhi-Firuzabad proposes to extend the storage capacity and invest in renewable generation units at different times, leading to a multistage stochastic problem. This model allows life-cycle constraints or decreasing technology efficiency to have an impact on results.

1.3 Contributions

Our contribution in this paper lies in four aspects. First, we propose an optimization model for a coupled management problem with both production and energy supply planning. We take into account the multistage structure of the problem, the uncertainties due to onsite renewable energy generation and binary variables modeling physical production constraints. In particular, we model shared resource constraints: a choice has to be made between different products at each time. Therefore, it is crucial, when reducing the problem to stage t with dynamic programming, to have visibility on the consequences of choosing a product at t. Second, we consider both on-demand supply with TOU pricing and in-advance energy purchasing. The latest brings complexity to the multistage problem with first-stage variables impacting the whole horizon costs. Third, we discuss multiple solution strategies based on well-known and new methodologies: a deterministic approach known as Model Predictive Control (MPC); Stochastic Dynamic Programming (SDP); and an approach solving linear multistage stochastic problems, Stochastic Dual Dynamic Programming (SDDP). Finally, as there does not exist an efficient algorithm to solve large mixed-integer multistage stochastic problems, we propose heuristic methods relying on the approximated cost-to-go function given by SDDP. We highlight the theoretical and practical limits of these solution strategies on numerical examples.

The remainder of the paper is laid out as follows. Section 2 introduces the problem formulation. We present in section 3 dynamic programming methods to solve multistage mixed-integer stochastic problems. Those methods being unsatisfactory for the problem at hand, we then proceed to detail different heuristics in section 4. Finally, section 5 presents numerical results.

1.4 Notations

To facilitate understanding, we go through some notation used in this paper. We denote $[a : b] := \{a, \dots, b\}$ the set of integers between a and b, and $[T] := [1 : T]$ the set of non-null integers smaller than T. Accordingly, $X_{[n]}$ denote the collection $X_{[n]} := \{X_i\}_{i \in [n]}$. Generally speaking, we denote the state variables x, the control variables u and the noise ξ. All random variables are in bold characters, further if $\boldsymbol{\xi}$ is a random variable then ξ denotes a realization of this variable. Finally, $\sigma(\xi_{[t]})$ represents the $\sigma-$algebra generated by $\{\xi_\tau\}_{\tau \in [t]}$.

2 A multistage stochastic formulation for joint production and energy planning

In this section, we present the mathematical formulation of our problem, presented in section 1.1. We first focus on the operational problem: daily operations the factory has to make. Note that, though we consider a specific production problem constructed from a practical industrial application, the proposed numerical approaches detailed in sections 3 and 4 can be adapted to other production problems.

We consider a factory owning solar panels and a battery. Thus, the energy supply is a mix of solar energy available $\boldsymbol{q_t}^{\text{PV}}$ (modeled as random variables), of charge $\boldsymbol{\phi_t}^+$ and discharge $\boldsymbol{\phi_t}^-$ from the battery, and of energy bought from the main grid $\boldsymbol{q_t}^{\text{grid}}$. Energy can be either bought in advance (*e.g.,* on a day-ahead market) or in real-time through industrial contracts with fixed prices. We decompose the energy bought from the grid $\boldsymbol{q_t}^{\text{grid}}$ into energy bought in advance v_t^{DA}, considered for now as a given parameter, plus energy bought during the day $\boldsymbol{v_t}^{\text{ID}}$. With these elements, we need to ensure that the energy supply exceeds the energy demand $\boldsymbol{q_t}^{\text{load}}$, leading to the following control constraints.

$$\boldsymbol{q_t}^{\text{PV}} + \boldsymbol{\phi_t}^- - \boldsymbol{\phi_t}^+ + \boldsymbol{q_t}^{\text{grid}} \geq \boldsymbol{q_t}^{\text{load}} \qquad \forall t \in [T], \tag{1a}$$
$$0 \leq \boldsymbol{\phi_t}^+ \leq \phi^+_{max} \qquad \forall t \in [T], \tag{1b}$$
$$0 \leq \boldsymbol{\phi_t}^- \leq \phi^-_{max} \qquad \forall t \in [T], \tag{1c}$$
$$\boldsymbol{q_t}^{\text{grid}} = v_t^{\text{DA}} + \boldsymbol{v_t}^{\text{ID}} \qquad \forall t \in [T], \tag{1d}$$
$$v_t^{\text{DA}}, \boldsymbol{v_t}^{\text{ID}} \geq 0 \qquad \forall t \in [T]. \tag{1e}$$

The energy demand $\boldsymbol{q_t}^{\text{load}}$ is shaped by the factory's production, derived from the quantities $(\boldsymbol{u_t^{ij}})_{t,i,j}$ of product j produced on machine i at time t. We also introduce binary variables, $(\boldsymbol{b_t^{ij}})_{t,i,j}$, assigning product j to machine i at time t, leading to the following set of constraint.

$$\sum_j \boldsymbol{b_t^{ij}} \leq 1 \qquad \forall i, t, \tag{1f}$$
$$\max_i \boldsymbol{b_t^{ij}} + \max_i \boldsymbol{b_t^{ij'}} \leq 1 \qquad \forall t, \forall (j, j') \in \mathcal{I}, \tag{1g}$$
$$u_t^{imin} \boldsymbol{b_t^{ij}} \leq \boldsymbol{u_t^{ij}} \leq u_t^{imax} \boldsymbol{b_t^{ij}} \qquad \forall i, j, t, \tag{1h}$$
$$\boldsymbol{q_t}^{\text{load}} = g(\boldsymbol{u_t^{ij}}, \boldsymbol{b_t^{ij}}) \qquad \forall t, \tag{1i}$$
$$\boldsymbol{b_t^{ij}} \in \{0, 1\} \qquad \forall i, j, t. \tag{1j}$$

At each time t, one machine can be assigned to one product at most (1f). $\mathcal{I}$ is the set of incompatible products, meaning a couple of products (j, j') belongs to $\mathcal{I}$ if they share resources. Therefore, they cannot be produced simultaneously (1g). Furthermore, production is bounded by machine capacities (1h). Finally, the function g gives the energy load induced by energy production (1i), we assume g linear.

Hence, the state of the system is described by the products and battery stocks. The stocks of products are modeled with state variables $(\boldsymbol{s_t^j})_{t,j}$. The demand at time t is modeled as a deterministic vector $(d_t^j)_{j\in J}$. Initial stocks are empty. Then the stock variables follow dynamic equations and bounding constraints given by

$$\boldsymbol{s_t^j} = \boldsymbol{s_{t-1}^j} - d_t^j + \sum_i \boldsymbol{u_t^{ij}} \qquad \forall t, j, \tag{2a}$$

$$\boldsymbol{s_t^j} \geq 0 \qquad \forall t, j, \tag{2b}$$

$$s_0^j = 0. \tag{2c}$$

Indeed, for each time t and product j, the factory has to satisfy a demand d_t^j, which is ensured by the positivity of stocks requirement (see eq. (2b)). Further, the quantity of energy stored in the battery, $(SOC_t)_t$, is also modeled as a state variable:

$$\boldsymbol{SOC_t} = \boldsymbol{SOC_{t-1}} - \frac{1}{\rho}\boldsymbol{\phi_t^-} + \rho\boldsymbol{\phi_t^+} \qquad \forall t, \tag{2d}$$

$$SOC_{min} \leq \boldsymbol{SOC_t} \leq SOC_{max} \qquad \forall t. \tag{2e}$$

For notational conciseness, we reduce control variables to vector $\boldsymbol{y_t} := (\boldsymbol{b_t}, \boldsymbol{u_t}, \boldsymbol{q_t^{\mathrm{grid}}}, \boldsymbol{\phi_t^+}, \boldsymbol{\phi_t^-})$ and state variables to vector $\boldsymbol{x_t} := (\boldsymbol{s_t}, \boldsymbol{SOC_t})$. Finally, the objective is to minimize total energy purchases *i.e.,* intra-day energy purchases v^{ID}.

$$V(x_0; v^{\mathrm{DA}}) := \min_{\boldsymbol{y_{[T]}}, \boldsymbol{x_{[T]}}} \quad \mathbb{E}\Big[\sum_{t=1}^{T} p_t^{\mathrm{ID}} \boldsymbol{v_t^{\mathrm{ID}}}\Big] \tag{3a}$$

$$s.t. \quad \text{eqs. (1) and (2),} \tag{3b}$$

$$\sigma(\boldsymbol{y_t}) \subset \sigma(\boldsymbol{q_{[t]}^{\mathrm{PV}}}) \qquad \forall t \in [T]. \tag{3c}$$

The last constraint eq. (3c), commonly known as non-anticipativity constraint, represents the information available when taking decision $\boldsymbol{y_t}$ at t. In particular, in this framework, we observe the random variable q_t^{PV} realization, before making decisions $\boldsymbol{y_t}$, with no knowledge of future random realizations from $t+1$ to T.

We now consider the strategic problem of choosing the best v^{DA} that minimizes day-ahead costs plus operational costs $V(x_0, v^{\mathrm{DA}})$. This can be done by introducing an initial time step $t = 0$ where such strategic variables are decided. This amounts to solving problem 4.

$$V(x_0) := \min_{v_t^{\mathrm{DA}} \geq 0} \quad \sum_{t=1}^{T} p_t^{\mathrm{DA}} v_t^{\mathrm{DA}} + V(x_0; v^{\mathrm{DA}}) \tag{4}$$

In the next section, we present solution methods for this problem based on Dynamic Programming.

3 Dynamic Programming approaches

Assuming that the noises are finitely supported, a multistage stochastic problem like Problem 3 can always be cast as a large-scale deterministic problem (see *e.g.,* [BL97]). However, the size of these deterministic equivalents is linear in the number of scenarios, which is often exponential in the horizon. A solution consists in compressing the information required to make a decision. To this end, we make a crucial stagewise independence assumption and turn to Dynamic Programming tools, presented here.

3.1 Stochastic Dynamic Programming

We consider a *controlled dynamic system*, that is a sequence of random vector $\boldsymbol{x}_{[T]}$ that follows a dynamic, affected by a sequence of noises $\boldsymbol{\xi}_{[T]}$. Those random vectors describe the state of the system across time, here product stocks $\boldsymbol{s_t}$ and battery energy level $\boldsymbol{SOC_t}$. Each noise ξ_t takes value in a finite set Ξ_t, and we denote $\Omega := \prod_{t \in [T]} \Xi_t$. We assume that these noises represent all the uncertainty in the problem at hand (here solar energy), with known probability distribution, resulting in a probability space $(\Omega, \mathcal{A}, \mathbb{P})$. We call *scenario* a sequence $\xi_{[T]}$ of realization of the noise at each time step.

We consider the Problem 3, parametrized by v^{DA} and restrained to sub-horizon $[t : T]$ from initial state x_{t-1}, and we denote its expected optimal value $V_t(x_{t-1}; v^{\mathrm{DA}})$. Then, with the stage-wise independence assumption, the Dynamic Programming principle ensures that the value functions follow the following recursive equations:

$$\hat{V}_t(x, \xi; v^{\mathrm{DA}}) = \min_{\substack{y_t \in \mathcal{Y}_t(x,\xi) \\ x_t \in \mathcal{X}_t(x, y_t, \xi)}} \quad \underbrace{p_t^{\mathrm{ID}} \boldsymbol{v_t}^{\mathrm{ID}}}_{\text{instantaneous cost}} + \underbrace{V_{t+1}(x_t; v^{\mathrm{DA}})}_{\text{cost-to-go}} \tag{5a}$$

$$V_t(x; v^{\mathrm{DA}}) = \mathbb{E}\left[\hat{V}_t(x, \boldsymbol{q_t}^{\mathrm{PV}}; v^{\mathrm{DA}})\right], \tag{5b}$$

$$V_{T+1}(x; v^{\mathrm{DA}}) = 0. \tag{5c}$$

For notational conciseness, we denote $\mathcal{Y}_t(x, \xi, v^{\mathrm{DA}})$ the feasible control set representing constraints eq. (1) depending on current state x, noise realization ξ, and strategic variables v^{DA}. Similarly, we denote $\mathcal{X}_t(x, u, \xi)$ the state set representing dynamics eq. (2) depending on previous state x, control u and noise ξ.

However, for any $x \in \mathcal{X}_{t-1}$, computing $V_t(x; v^{\mathrm{DA}})$ requires full knowledge of V_{t+1}. With continuous state, it is usually impossible. Therefore, to accommodate for inexact value functions, we introduce the bellman operators which generalize eq. (5) so that the dynamic equations hold for any given function R approximating the cost-to-go V_{t+1}. The backward operator $\mathcal{B}_t$, defined in eq. (6a),

$$\text{Backward operators} \quad \begin{cases} \hat{\mathcal{B}}_t(R) : x, \xi \mapsto \min\limits_{\substack{y_t \in \mathcal{Y}_t(x,\xi) \\ x_t \in \mathcal{X}_t(x, y_t, \xi)}} p_t^{\mathrm{ID}} \boldsymbol{v}_{\boldsymbol{t}}^{\mathrm{ID}} + R(x_t), \\ \mathcal{B}_t(R) : x \mapsto \mathbb{E}\left[\hat{\mathcal{B}}_t(R)(x, \boldsymbol{q}_{\boldsymbol{t}}^{\mathrm{PV}})\right], \end{cases} \tag{6a}$$

returns an approximation, at a given state x, of the cost-to-go V_t starting from time t, given an approximation R of the cost-to-go starting from time $t+1$. Thus, given a discretization of each state space X_t, and an interpolation method we can, recursively, compute an approximation of every cost-to-go function see algorithm 1.

Algorithm 1: Stochastic Dynamic Programming

Input : x_0, discretization grids $\mathcal{X}_t^D$, interpolation operator.
Output : approximated value function $\tilde{V}_t$
$\tilde{V}_{T+1} = 0$.
for $t : T \to 1$ **do**
  for $x_{t-1}^D \in \mathcal{X}_{t-1}^D$ **do**
    `// We discretize` $\mathcal{X}_t$
    for $\xi_t \in \Xi_t$ **do**
      Solve the one-stage deterministic optimization problem:
        $\tilde{V}_t(x_{t-1}^{\mathrm{D}}, \xi_t; v^{\mathrm{DA}}) = \hat{\mathcal{B}}_t(\tilde{V}_{t+1})(x_{t-1}^{\mathrm{D}}, \xi_t; v^{\mathrm{DA}})$.
    $\tilde{V}_t(x_{t-1}^{\mathrm{D}}; v^{\mathrm{DA}}) = \sum\limits_{\xi_t \in \Xi_t} \pi_{\xi_t} \tilde{V}_t(x_{t-1}^{\mathrm{D}}, \xi_t; v^{\mathrm{DA}})$; `// expected value`
  Define $\tilde{V}_t$ for any $x \in \mathcal{X}_{t-1}$ by interpolation on $\left\{(x_{t-1}^D, \tilde{V}_t(x_{t-1}^D; v^{\mathrm{DA}}))\right\}_{x_{t-1}^D \in \mathcal{X}_{t-1}^{\mathrm{D}}}$.

We then define in eq. (6b) the forward operator, which returns the optimal next state x_t, given a starting state x, a noise ξ and an approximation R of the cost-to-go from $t+1$. Note that, in practice, computing $\hat{\mathcal{B}}_t(R)(x, \xi)$ or $\hat{\mathcal{F}}_t(R)(x, \xi)$ consists in solving the same deterministic problem. Nevertheless, if the backward operator is well-defined, the forward operator requires a choice if the optimal solution is not unique. To be completely rigorous, we should say that a forward operator defines a selection of the optimal solution set.

$$\hat{\mathcal{F}}_t(R) : x, \xi \mapsto y_t^\star, x_t^\star \in \operatorname*{arg\,min}_{\substack{y_t \in \mathcal{Y}_t(x,\xi) \\ x_t \in \mathcal{X}_t(x, y_t, \xi)}} p_t^{\mathrm{ID}} \boldsymbol{v}_{\boldsymbol{t}}^{\mathrm{ID}} + R(x_t) \tag{6b}$$

Dynamic Programming is a powerful tool as the multistage problem considers $(|\Xi|^T)$ scenarios and turns the exponential complexity in the horizon T into a linear one. However, it is limited by what is known as the *curse of dimensionality*. Indeed, we have to solve, for each time step, $|\mathcal{X}_t|.|\Xi|$ problem. A discretization of $\mathcal{X}_t$ usually requires a number of points exponential in the dimension of $\mathcal{X}_t$. Thus, in practice, Dynamic Programming cannot be used for states with more than 5 dimensions.

Remark 1. *In order to represent the strategic problem as a stochastic dynamic system, we need consider an extended state* (x_t, v^{DA})*, where* v^{DA} *is decided at* $t = 0$ *and then carried on, as part of the state, from stage to stage by the dynamics of the system. This extension increases the dimension of the system from* $J+1$ *to* $J+1+T$*, making algorithm 1 computationally intractable as typical* T *is at least* 24.

When $J \leq 3$ algorithm 1 can be reasonably used to address Problem 3. However, computation time is still high as we solve $\mathcal{O}(T.|\mathcal{X}_t^D|.|\Xi|)$ MILP. Thus, we present another algorithm, exploiting sampling methods, in the next section.

3.2 Stochastic Dual Dynamic Programming (SDDP)

To counteract the dynamic programming computational issues, a class of *Trajectory Following Dynamic Programming* (TFDP) algorithms (see [FL22] for recent overviews) has been developed. The crux of these algorithms is to iterate between forward phases that compute state trajectories, and backward phases that improve cost-to-go estimations. More specifically, in the forward phase of a TFDP algorithm, a state trajectory is computed using the current cost-to-go estimations. Then in a backward phase, the cost-to-go estimations are refined around the state trajectory computed in the forward phase. These approximations are given as the maximum of elementary functions called *cuts*.

For linear multistage stochastic problems with stagewise independence, the SDDP algorithm [PP91] has proven to be an efficient tool, widely used in the energy community in particular for long-term hydro-management. It is the most well-known and studied example of TFDP algorithm, relying on Benders' cut obtained through linear programming duality, assuming the problem is convex and continuous. In line with SDDP, the Stochastic Dual Dynamic Integer Programming (SDDiP), [ZAS19], assumes that all state variables are binary, that there exists some continuous recourse ensuring relatively complete recourse assumption, and derives specific linear cuts. As one can always represent bounded integer variables, and approximate continuous variables, through binaries, the algorithm is theoretically applicable for a large number of settings, including ours, but is limited in practice as each step requires solving a MILP, and as the convergence is generally slow.

Another TFDP algorithm, the Mixed Integer Dynamic Approximation Scheme (MIDAS) (see [PWB20]) assumes the monotonicity of the cost-to-go functions and uses piecewise constant cuts to approximate them. Finally, the Stochastic Lipschitz Dynamic Programming (see [ACF22]), simply assumes Lipschitz regularity of the cost-to-go functions and uses reverse norm cuts. SDDiP, MIDAS and SLDP might be applicable to the industrial microgrid setting but are generally slow to converge without additional, problem-specific, cuts. However, the subject is an active field of research: variants and enhancements of those algorithms are frequently published (*e.g.,* [FR22; QGK23]). Unfortunately, to the best of our knowledge, there is no off-the-shelf implementation of efficient TFDP algorithms for mixed-integer stochastic programs.

Therefore, we consider the continuous relaxation of Problem 3, and adapt the tools in section 3.1 for the continuous relaxation, using exponent r to indicate the problem at hand is relaxed. It is the same problem as Problem (4) but we assume all binary variables are in $[0,1]$ instead of $\{0,1\}$, represented by $\boldsymbol{y_t^r} \in \mathcal{Y}_t^r(\boldsymbol{x_{t-1}}, \boldsymbol{q_t}^{\mathrm{PV}}; v^{\mathrm{DA}})$.

Leveraging the convexity of the relaxed Problem 3, the SDDP algorithm 2, approximates each V_{t+1}^r as a maximum of affine functions. More precisely, at iteration k, we first compute a trial trajectory $(x_t^k)_{t\in[T]}$. Then, in the backward phase, we can compute $\mathcal{B}_t^r(V_{t+1}^{r,k})$ by solving $|\Xi_t|$ linear problems. Linear programming duality

Algorithm 2: Stochastic Dual Dynamic Programming

// Initialization

$k = 0$, $V_t^{r,0} = LB$, v^{DA}.

for $k : 0, \dots$ **do**

  Simulate a scenario $\{\xi_t^k\}_{t\in[T]}$.

  // Forward phase

  $x_0^k = x_0$.

  for $t : 1 \to T$ **do**

    $y_t^k, x_t^k = \hat{\mathcal{F}}_t^r(V_t^{r,k})(x_{t-1}^k, \xi_t^k; v^{\mathrm{DA}})$.

  // Backward phase

  $V_{T+1}^{r,k} = 0$

  for $t : T \to 1$ **do**

    // Cut computation

    for ξ *realization of* $\boldsymbol{\xi_t}$ **do**

      Solve $\hat{\mathcal{B}}_t^r(V_{t+1}^{r,k})(x_{t-1}^k, \xi; v^{\mathrm{DA}})$ and obtain coefficients $\hat{\alpha}_t^k(\xi)$ and $\hat{\beta}_t^k(\xi)$ such that:

$$\hat{\alpha}_t^k(\xi)^T x + \hat{\beta}_t^k(\xi) \leq \hat{\mathcal{B}}_t^r(V_{t+1}^{r,k})(x, \xi; v^{\mathrm{DA}}) \quad \forall x.$$

    Define $\alpha_t^k = \mathbb{E}\left[\hat{\alpha}_t^k(\boldsymbol{\xi_t})\right]$ and $\beta_t^k = \mathbb{E}\left[\hat{\beta}_t^k(\boldsymbol{\xi_t})\right]$.

    Define $V_t^{r,k} : x \mapsto \max_{\kappa\leq k}(\alpha_t^{\kappa\,T} x + \beta_t^\kappa)$.

yields a sub-gradient of $\mathcal{B}_t^r(V_{t+1}^{r,k+1})$ at x_{t-1}^{k+1}, which in turn defines an affine function which underestimates $\mathcal{B}_t^r(V_{t+1}^{r,k+1}) \leq \mathcal{B}_t^r(V_{t+1}^r) = V_t^r$. In particular, at iteration k, the approximate cost-to-go functions $V_t^{r,k}$ are given as a maximum of affine cuts, *i.e.*, $V_t^{r,k}(x) = \max_{\kappa\leq k}\{\alpha_t^\kappa + \beta_t^\kappa x\}$.

Recall that, given any approximated cost-to-go function, the forward Bellman operator (see section 3.1), produces a state-based feedback, satisfying in particular the binary constraints. Thus, it seems natural to use the functions $V_t^{r,K}$ as approximated cost-to-go, leading to a strategy through the forward operators $\hat{\mathcal{F}}_t(V_t^{r,K})$. The main limit of this approach is that we are quite greedy in the way we repair the binary constraints. Indeed, $V_t^{r,K}$ does not account for binary constraints, and the forward operator only considers their impact on one time-step. In particular in the problem at hand (with shared resource constraints), SDDP approximated cost-to-go functions do not capture the necessity to make a choice between two products in the future. Therefore, a decision at t leading to infeasibility in the future, can have a finite SDDP estimated cost-to-go, and be selected by the forward operator. We illustrate the limit of this approach in the following toy example.

Example 1 (Limit of continuous relaxation.)**.** *Consider a production unit that produces two products $j = A, B$, over $T = 2$ time steps and one machine. The shared resource constraint, modeled through binary variables b_t^j, implies that we must decide which product to produce at $t = 1$, and which at $t = 2$. We look for the production plan minimizing costs while satisfying a demand $D = 1$ in both products at the end of the horizon. The problem is formalized as follows.*

$$
\begin{aligned}
\min \quad & 3u_1^A + 2u_1^B + (u_2^A + u_2^B) && && \text{(7a)}\\
s.t \quad & u_1^j + u_2^j \geq D && j = A, B, && \text{(7b)}\\
& 0 \leq u_t^j \leq 2b_t^j && j = A, B \quad t = 1, 2, && \text{(7c)}\\
& b_t^A + b_t^B \leq 1 && t = 1, 2, && \text{(7d)}\\
& b_t^j \in \{0, 1\},\ u_t^j \geq 0 && j = A, B \quad t = 1, 2. && \text{(7e)}
\end{aligned}
$$

For the true problem, it is optimal to produce B in the first period and A in the second period, resulting in an optimal cost of 3*. However, in the continuous relaxation of Problem* (7)*,* $b_t^j \in [0, 1]$*, and producing both products at the same time is allowed. For instance, producing both products at time* $t = 2$ *(with* $b_2^A = b_2^B = 0.5$*) is admissible for the relaxed problem, yielding an optimal cost of* 2*.*

Let V_2^r *be the relaxed cost-to-go function given by:*

$$
\begin{aligned}
V_2^r(u_1^A, u_1^B) = \min_{u_2^A, u_2^B, b_2^A, b_2^B} \quad & u_2^A + u_2^B && && \text{(8a)}\\
s.t \quad & u_1^j + u_2^j \geq D && j = A, B, && \text{(8b)}\\
& 0 \leq u_2^j \leq 2b_2^j && j = A, B, && \text{(8c)}\\
& b_2^A + b_2^B \leq 1, && && \text{(8d)}\\
& b_2^j \geq 0,\ u_2^j \geq 0 && j = A, B. && \text{(8e)}
\end{aligned}
$$

Now, using the cost-to-go approximation V_2^r *to determine optimal decisions of the mixed-integer problem at* $t = 1$*, we solve:*

$$
\begin{aligned}
\min_{u_1^A, u_1^B, b_1^A, b_1^B} \quad & 3u_1^A + 2u_1^B + V_2^r(u_1^A, u_1^B) && && \text{(9a)}\\
s.t \quad & b_1^A + b_1^B \leq 1, && && \text{(9b)}\\
& 0 \leq u_1^j \leq 2b_1^j && j = A, B, && \text{(9c)}\\
& b_1^j \in \{0, 1\} && j = A, B. && \text{(9d)}
\end{aligned}
$$

Note that, when solving Problem9, we make decisions at $t = 1$ *considering the cost impact at* $t = 2$*, but not knowing what decisions are attached to this cost. In dynamic programming, infeasibility is supposed to be propagated through costs: in this example, with the real cost-to-go function,* $V_2(0, 0) = +\infty$ *and the solution* $u_1^A = u_1^B = 0$ *would never be chosen. However, if we use the relaxed cost-to-go function, the infeasible solution* $u_1^A = u_1^B = 0$ *has a cost* $0 + V_2^r(0, 0) = 2$ *and is chosen rather than the optimal solution* $u_1^A = 1; u_1^B = 0$*, whose cost is* $2 + V_2^r(0, 1) = 3$*.*

We move on to present heuristics in section 4, and address this particular limit in section 4.4 through a look-ahead heuristic that consider more than one time-step.

4 Heuristics for multistage problems

So far, we have presented a stochastic algorithm with unreasonable computational time and a stochastic algorithm solving a continuous relaxation of our problem. Those are exact methods, but will not allow us to solve the problem in a satisfactory manner. Could we come up with heuristics taking into account uncertainties, using SDDP, and solving mixed-integer problems such as ours? In this section, we present different heuristics, either relaxing information constraints *i.e.,* deterministic, or relaxing integrity to a point.

4.1 An Expected Value (EV) corrected heuristic

One of the challenges is to take into account random variables. A common simplification consists in reducing the problem to its deterministic version, by replacing the random variable with our current best estimation. We are then in the anticipative framework which consists in assuming we can look into the future and know the noises realization *i.e.,* relaxing constraint 3c. More precisely, solving the anticipative problem, given a scenario $q^{\mathrm{PV}}_{[T]}$, returns a solution perfectly adapted to this scenario, of optimal value $V^{ant}(x_0, q^{\mathrm{PV}}_{[T]})$. Then, the Expected Value solution amounts to solving the anticipative problem, given the expected scenario $\overline{q}^{PV}_{[T]}$.

However, we are not in a complete recourse setting, meaning that the deterministic production and energy plan computed is not necessarily admissible. Therefore, a first heuristic consists in computing the deterministic solution fixing part of control variables, and then, adjusting the rest of the variables to actual random variable realization. In our particular microgrid problem, we fix production variables and then adjust energy flows to actual solar energy produced. We opt for a simple strategy described in fig. 2.

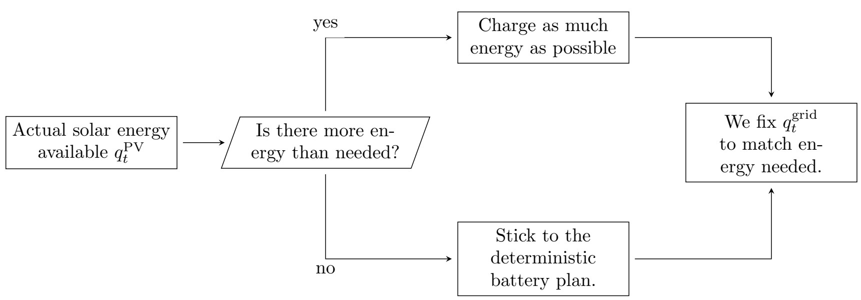


Figure 2: Corrected EV heuristic algorithmic scheme

This strategy has no flexibility, which is needed in a system subjected to uncertainties. It serves as a benchmark for stochastic solutions.

4.2 Model Predictive Control

To add flexibility to the previous approach, we present the Model Predictive Control (MPC) approach, as a first adaptive approach. To use MPC we need some *forecast* methodology, that takes available information to predict the values of the random variables $\{\boldsymbol{q_t}^{\mathrm{PV}}\}_{t\in[T]}$. The algorithm then consists in solving successive deterministic sub-problems (see algorithm 3). Step after step, it applies the decision of the first control obtained, reveals the realization of the next random variable, and recomputes all other decisions, updating forecasted values if possible.

Algorithm 3: Model Predictive Control

Input : x_0, initial forecast $\{q^{\text{PV}}_{\tau,0}\}_{\tau\in[T]}$.

$$v^{\text{DA}} = \quad \arg\min \sum_{t=1}^{T} p^{\text{DA}}_t v^{\text{DA}}_t + V^{ant}(x_0, q^{\text{PV}}_{[T],0}; v^{\text{DA}})$$

for $t : 1 \to T$ **do**

Update forecasted values $\{q^{\text{PV}}_{\tau,0}\}_{\tau\in[T]}$.

$$\begin{aligned} y^*_t, \dots, y^*_T = \quad & \arg\min \quad \sum_{\tau=t}^{T} p^{\text{ID}}_t v^{\text{ID}}_t \\ & \text{s.t.} \quad y_\tau \in \mathcal{Y}_\tau(x_{\tau-1}, q^{\text{PV}}_{\tau,0}; v^{\text{DA}}) && \forall \tau \in [t:T], \\ & \qquad\; x_\tau \in \mathcal{X}_\tau(x_{\tau-1}, y_\tau, q^{\text{PV}}_{\tau,0}) && \forall \tau \in [t:T]. \end{aligned}$$

$x_t \in \mathcal{X}(x_{t-1}, y^*_t, q^{\text{PV}}_{t,0})$

As long as we can get a solution to the MILPs in a reasonable time, MPC is an easy option to implement. However, this method yields no performance guarantee, and does not really take randomness into account, as the solution is computed for a single possible realization, but simply recomputes the solution as more information becomes available. Consequently, the quality of the solution provided by MPC depends mainly on the quality of the forecasted values, the flexibility of the problem and the sensitivity of the problem to uncertainty.

4.3 2-stage stochastic programming

The strategic design problem (P) balances the design cost $\sum_t p^{\text{DA}}_t v^{\text{DA}}_t$ and the operational cost $V(x_0; v^{\text{DA}})$. The 2-stage stochastic programming consists in relaxing the non-anticipativity constraint for all operational decisions. Hence, the design problem becomes a two-stage stochastic program, where the first stage decision is the strategic decision v^{DA} and the recourse are the operational decisions.

$$\min_{v^{\text{DA}}\in\mathbb{R}^T_+} \qquad \sum_t p^{\text{DA}}_t v^{\text{DA}}_t + \mathbb{E}\left[\hat{V}^{ant}(x_0, \boldsymbol{q}^{\text{PV}}_{[\boldsymbol{T}]}; v^{\text{DA}})\right] \tag{10}$$

However, computing the exact value of $\mathbb{E}\left[\hat{V}^{ant}(x_0, \boldsymbol{q}^{\text{PV}}_{[\boldsymbol{T}]}; v^{\text{DA}})\right]$ would require to solve a deterministic operational problem for each possible scenario $q^{\text{PV}}_{[T]} \in \Omega$. There is usually far too many scenarios to consider, thus, we resort to Sample Average Approximation, which is the 2-stage extension of Monte Carlo methods. We draw S_{MC} scenarios, and obtain the following 2-stage formulation:

$$
\begin{aligned}
V^{2S_{MC}}(x_0) \quad &:= \min_{v^{\mathrm{DA}} \in \mathbb{R}_+^T} \min_{(x_t^s, y_t^s)} \quad \sum_t p_t^{\mathrm{DA}} v_t^{\mathrm{DA}} + \sum_{s=1}^{S_{MC}} \frac{1}{S_{MC}} \left(\sum_{t=1}^{T} p_t^{\mathrm{ID}} v_{t,s}^{\mathrm{ID}} \right) && \text{(11a)} \\
\text{s.t.} \quad & x_t^s \in \mathcal{X}_t(x_{t-1}^s, y_t^s, q_{t,s}^{\mathrm{PV}}) \qquad \forall t \in [T], \forall s \in [S_{MC}], && \text{(11b)} \\
& y_t^s \in \mathcal{Y}_t(x_{t-1}^s, q_{t,s}^{\mathrm{PV}}, v^{\mathrm{DA}}) \qquad \forall t \in [T],\ \forall s \in [S_{MC}]. && \text{(11c)}
\end{aligned}
$$

All the approaches presented in this section up to this point relax non-anticipativity constraints but keep binary constraints by solving MILPs. In section 3.2, we saw that SDDP solves problem3 with non-anticipativity constraints but relaxing binary constraints. We now look for a trade-off between information relaxation and integrity relaxation.

4.4 Look-ahead heuristic

Were the forward operator (see eq. (6b)) to have more visibility on the future variable possibilities (or impossibilities), we have the intuition that the algorithm would perform better. Indeed, as it is defined, the operator takes the best decision possible at t by optimizing a one-stage problem minimizing the current cost at t plus an approximate cost-to-go function from $t+1$. Details of the problem complexity are thus only represented over one stage, and the impact of decision at time t on the next stage should all be taken into account by the approximate cost-to-go function.

To have a better representation of the problem, we can consider τ-stage problems with a final cost-to-go function $\tilde{V}_{t+\tau}$ instead of one-stage problems (with final cost-to-go function $\tilde{V}_{t+1}$). More precisely we define a τ-look-ahead Bellman operator $\mathcal{B}_t^\tau$ as:

$$
\begin{aligned}
\hat{\mathcal{B}}_t^\tau(R):\ x, \xi \mapsto \min_{y_t, x_t} \quad & p_t^{\mathrm{ID}} v_t^{\mathrm{ID}} + \min_{(\boldsymbol{x}_{t'}, \boldsymbol{y}_{t'})_{t' \in [t+1:t+\tau]}} \mathbb{E}\left[\sum_{t'=t+1}^{t+\tau} p_{t'}^{\mathrm{ID}} \boldsymbol{v}_{\boldsymbol{t'}}^{\mathrm{ID}} + R(\boldsymbol{x}_{t+\tau}) \right] && \text{(12a)} \\
\text{s.t.} \quad & x_t \in \mathcal{X}_t(x, y_t, \xi), && \text{(12b)} \\
& y_t \in \mathcal{Y}_t(x, \xi; v^{\mathrm{DA}}) && \text{(12c)} \\
& \boldsymbol{x}_{\boldsymbol{t'}} \in \mathcal{X}_{t'}(\boldsymbol{x}_{\boldsymbol{t'-1}}, \boldsymbol{y}_{\boldsymbol{t'}}, \boldsymbol{q}_{\boldsymbol{t'}}^{\mathrm{PV}}) \qquad t' \in [t+1 : t+\tau], && \text{(12d)} \\
& \boldsymbol{y}_{\boldsymbol{t'}} \in \mathcal{Y}_{t'}(\boldsymbol{x}_{\boldsymbol{t'-1}}, \boldsymbol{q}_{\boldsymbol{t'}}^{\mathrm{PV}}; v^{\mathrm{DA}}) \qquad t' \in [t+1 : t+\tau], && \text{(12e)} \\
& \sigma(\boldsymbol{u}_{\boldsymbol{t'}}) \subset \sigma(\boldsymbol{q}_{\boldsymbol{[t+1:t']}}^{\mathrm{PV}}) \qquad t' \in [t+1 : t+\tau]. && \text{(12f)} \\
\mathcal{B}_t^\tau(R):\ x \quad \mapsto \quad & \mathbb{E}\left[\hat{\mathcal{B}}_t^\tau(R)(x, \boldsymbol{q}_{\boldsymbol{t}}^{\mathrm{PV}}) \right] && \text{(12g)}
\end{aligned}
$$

In this setting, the first-stage decisions are optimized knowing the impact they have on the next $\tau - 1$ stages, thanks to eqs. (12b) to (12f), and a cost-to-go function R from $t+\tau+1$. However, the $\tau-$stage decisions are taken without any visibility on the future except a given cost-to-go function. For this reason, when solving each $\tau-$stage problem $\mathcal{B}_t^\tau(R_{t+\tau+1})(x_{t-1})$, we only store the first-stage variables y_t and then move along to the next sub-problem $\mathcal{B}_{t+1}^\tau(R_{t+\tau+2})(x_t)$.

In a sense, we allow the operators to look ahead of time to choose their decision at t, and call this method the look-ahead heuristic. We associate to the backward operator $\hat{\mathcal{B}}_t^\tau$ a forward operator $\hat{\mathcal{F}}_t^\tau(R) : \mathcal{X}_{t-1} \times \Xi_t \to \mathcal{X}_t \times \mathcal{Y}_t$ which returns $x_t^\star, y_t^\star$ depending on current state x and noise realization ξ.

For clarity, we explicitly give the 2-look-ahead Bellman operator:

$$\begin{aligned}
\hat{\mathcal{B}}_t^2(R)(x,\xi) = & \min_{\substack{x_t,(x_{t+1}^s)_{s\in|\Xi_{t+1}|}\\ y_t,(y_{t+1}^s)_{s\in|\Xi_{t+1}|}}} p_t^{\mathrm{ID}} v_t^{\mathrm{ID}} + \sum_s \mathbb{P}(\boldsymbol{q}_{\boldsymbol{t+1}}^{\mathrm{PV}} = q_{t+1,s}^{\mathrm{PV}}) \Big[p_{t+1}^{\mathrm{ID}} v_{t+1,s}^{\mathrm{ID}} + \mathcal{R}(x_{t+1}^s) \Big] && \text{(13a)} \\
& \text{s.t.} \quad x_t \in \mathcal{X}_t(x, y_t, \xi), \\
& \qquad\quad y_t \in \mathcal{Y}_t(x, \xi; v^{\mathrm{DA}}), \\
& \qquad\quad x_{t+1}^s \in \mathcal{X}_{t+1}(x_t, y_{t+1}^s, q_{t+1,s}^{\mathrm{PV}}) && \forall s \in |\Xi_{t+1}|, \\
& \qquad\quad y_{t+1}^s \in \mathcal{Y}_{t+1}(x_t^s, q_{t+1,s}^{\mathrm{PV}}; v^{\mathrm{DA}}) && \forall s \in |\Xi_{t+1}|, \\
\mathcal{B}_t^2(R)(x) = & \ \mathbb{E}\left[\hat{\mathcal{B}}_t^2(R)(x, \boldsymbol{q_t}^{\mathrm{PV}})\right]. && \text{(13b)}
\end{aligned}$$

Note that this 2-look-ahead Bellman operator considers the exact cost at t and $t+1$, and uses R as an estimation of expected cost-to-go from $t+2$ to T. In particular, due to the new information, we must consider as many decisions y_{t+1}^s as there are realizations for the random variable $\boldsymbol{q}_{\boldsymbol{t+1}}^{\mathrm{PV}}$.

Combining these new operators with the approximated cost-to-go functions computed by SDDP (see section 3.2), we get a heuristic where the non-anticipativity constraints hold at any time, and the integrity constraints are kept on τ time steps. Unfortunately, increasing the look-ahead horizon *i.e.,* τ, greatly increases the complexity of the sub-problems we solve. For instance, with $|\Omega_t| = 10$, the backward operator $\hat{\mathcal{B}}_t^\tau$ at t solves a problem with $10^{\tau-1}$ times more variables than $\hat{\mathcal{B}}_t$.

5 Numerical results

We now present a study case from our industrial partner on which we evaluate the numerical methods presented above. In section 5.1 we detail the study case, intraday results, given in section 5.2, show that the MPC method is most adapted to our study, it is then used for the day-ahead problem in section 5.3 where SDDP shows its advantages.

5.1 Study Case

The problem described in section 5.1 is motivated by a cement factory in South Korea. We solve the problem for hourly planning on one day, with $T = 24$ time steps. In the Republic of Korea, electricity rates are fixed for the industry and depend on different time slots and seasons. We took the rates given by the Korea Electricity Power Corporation website [Newa] and thus obtained $\{p_t^{\mathrm{ID}}\}_{t\in[T]}$. We consider that buying energy in advance is cheaper and fix the day-ahead rates to 90% of the real-time rates.

Then we collect solar irradiance data on [Newb]. From this data, we use a forecast algorithm to predict a daily solar energy generation for a park of capacity $C^{PV} \in \{2, 4, 8, 12\}$ in MWc. The model is trained on the last 72 hours data to produce generation scenarios over the next 24 hours. From this model we estimate, at each time step t, 9 quantiles. We finally assume that the noise is stagewise independent, leading to 9^T scenarios.

The factory owns $I = 3$ mills and produces $J = 3$ different types of cement. Production bounds are given by the factory. An analysis of the factory's data leads us to model a mill's energy consumption, on the range $[u_t^{imin}, u_t^{imax}]$, as an affine function of its cement production (1i).

We study the impact of three different battery sizes, proportional to the installed renewable capacity: SOC_{max} is equal to the maximum quantity of energy the solar panels can produce in 0.5, 3 or 6 hours. For example, on a solar park of 4 MWc, we consider three battery capacities: 2, 12 or 24 MWh. We also fix ϕ^+_{max} and ϕ^-_{max} to a quarter of the battery's capacity per time-step and the efficiency factor ρ to 0.9.

5.2 Intraday results

In this section, we present and analyze the results obtained when solving problem (3) on instances in which energy can only be bought in real-time, which is equivalent to fixing $v_t^{\mathrm{DA}} = 0$ for all t. Further, we only consider a demand at the end of the day: $d_t^j > 0$ only for $t = T$.

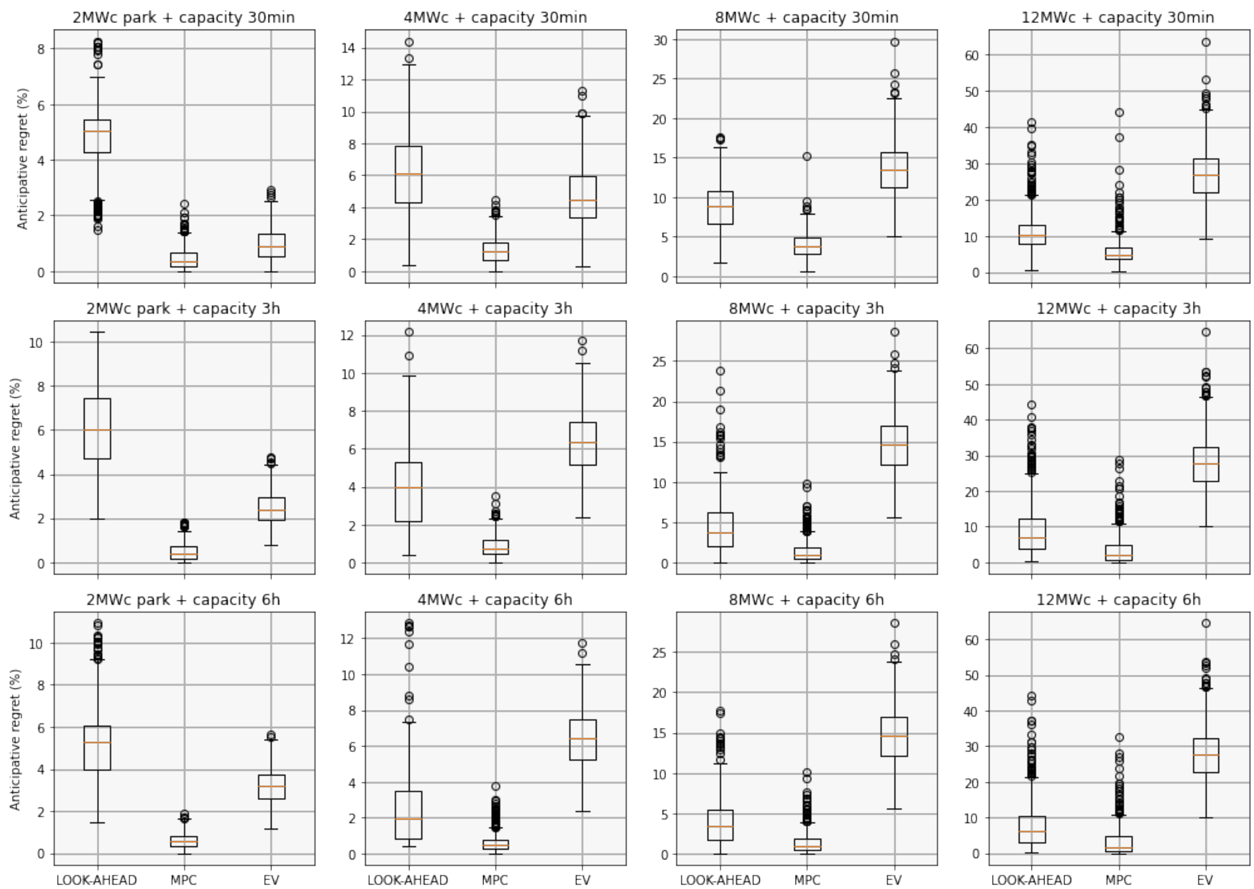


Figure 3: Anticipative regret (AR) in percentage for different solar park capacity and ESS capacity: increasing solar energy (and thus variability) from left to right, and increasing battery storage capacity (proportional to solar energy available) from top to bottom.

On a given day, for various renewable size ($C^{PV} \in \{2, 4, 8, 12\}$) and battery sizing ($SOC_{max}$ represents $0.5, 3$ or 6 hours of maximum renewable production), we test the different strategies, evaluating them over 500 common scenarios drawn from our statistical model. More precisely, we compare:

1. the elementary strategy, described in section 4.1, which solves the EV problem abd then adapt energy variables following a deterministic procedure as noises are revealed;
2. the MPC strategy, see section 4.2, which consists in solving deterministic sub-problems at each stage, with updated information, to adjust the solution trajectory accordingly;
3. and the Look-Ahead (LA), with $\tau = 2$, explained in section 4.4, strategy which computes a solution with dynamic programming using an under-approximation of future costs given by SDDP.

Each strategy yields a noise-based policy which, depending on a scenario, computes a trajectory of the system. To evaluate a strategy's performance over a given scenario, we define the *anticipative regret* of admissible noise-based policy π, on a scenario $\xi_{[T]}$, as the relative gap between its cost and the anticipative lower bound:

$$AR^{\pi}(\xi_{[T]}) = \frac{\hat{V}^{\pi}(x_0, \xi_{[T]}; v^{\mathrm{DA}}) - V^{ant}(x_0, \xi_{[T]}; v^{\mathrm{DA}})}{|V^{ant}(x_0, \xi_{[T]}; v^{\mathrm{DA}})|}. \tag{14}$$

In fig. 3 we report the anticipative regret of each strategy. The results clearly show MPC's superiority in these instances. On the one side, the EV heuristic yields unsatisfactory results in comparison to MPC: its expected anticipative regret is always higher, and its expected cost as well. Further, except on the first column, which corresponds to instances with few uncertainties (*i.e.,* a solar park of 2MW), and the first instance of the second column (a more uncertain instance but with a small battery), the EV heuristic performs worse than the look-ahead heuristic. As uncertainties grow (from left to right), the costs of the EV heuristic are farther and farther away from the anticipative lower bound, showing that a purely deterministic procedure is not relevant to our problem.

On the other side, the look-ahead heuristic, properly taking uncertainties into account with a stochastic procedure, but relaxing some integrity constraints, does not perform as well as MPC. Indeed, the latter, adjusting the solution trajectory to uncertainties, yields solutions close to their anticipative lower bound: even for the most volatile instances (*i.e.,* the ones with a solar park of 12MWc, all on fig. 3's fourth column), the anticipative regret is lower than 5% and in most cases insignificant. These performances can be explained by the problem structure: the uncertainty source does not impact significantly future costs, in the case of solar energy variations at t, MPC foresees the cost impact and adapts accordingly. Furthermore, for industrial problems with renewable generation, we confirm the necessity of installing an ESS to make the system flexible. In fig. 4, we plot the optimal expected cost of the various methods on instances with growing ESS capacity. Clearly, the expected optimal expected cost decreases as the ESS capacity increases, although the marginal impact of the ESS capacity is decreasing.

Whereas MPC results are better, we call attention to its limits: on table 1 we can see that MPC takes longer in computation time than the look-ahead heuristic, even more so on instances with the most variability. In these instances, it remains reasonable (a few seconds per problem at the most for an hour step time problem), but with larger instances, and more constraints, it could be unsuitable. Note that SDDP converges after only a 100 iterations, taking approximately $250s$ per instance.

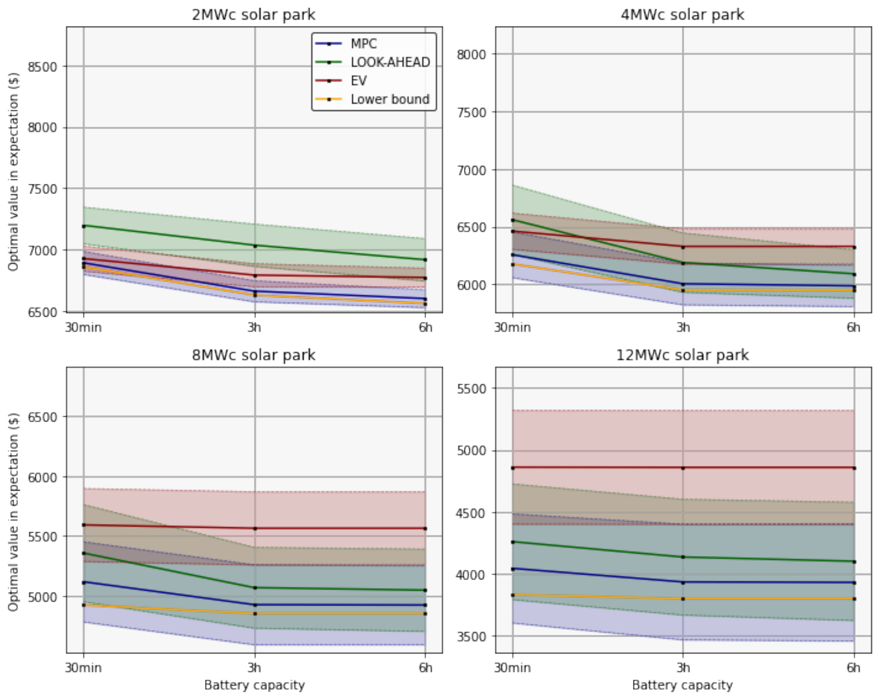


Figure 4: Expected value of strategies with 95% confidence interval.

5.3 Day-ahead results

We now consider the full Problem 4 with strategic and operational decisions. In particular, we consider an initial time step ($t = 0$), where the industrial buys in advance energy quantities for the whole horizon. To our knowledge, this type of contract does not exist yet in South Korea, but it could be interesting for the regulator to encourage certain consumption schemes. It can also model the access to energy markets for large consumers or consumers aggregated through virtual power plants. We fix the in-advance prices at 90% of intra-day prices.

The problem can be decomposed into two parts: first a strategical problem with variable v^{DA}, then an operational sub-problem, parametrized by v^{DA}. Our intuition is that a deterministic method might not be flexible enough because first-stage decisions impact the whole horizon. Note that the parametrized problem 3 corresponds to the intraday problem we solve in section 5.2. We saw that the most efficient method to solve problem 3 is MPC. In this section, we determine through different methods the best strategical decision v^{DA} and then run MPC on the parametrized operational problem.

We assume that the demand is only positive at the end of the day $d_T^j > 0$ and we test various renewable sizes ($C^{PV} \in \{2, 4, 8, 12\}$). In section 5.2, we tested different battery size, and results showed that extending the battery capacity, to a certain point, improves costs and the system flexibility. Consequently, we now fix the battery capacity to 3 hours of maximum renewable production.

To optimize v^{DA}, we test 3 methods evaluated over 1000 common scenarios:

SOC_{max}	0.5h			3h			6h		
C^{PV} (MWc)	MPC	L-A	SDDP	MPC	L-A	SDDP	MPC	L-A	SDDP
2	21	6.5	277	12	7.6	268	25	20	262
4	26	8.0	213	4.4	2.9	225	38	18	238
8	254	11	249	136	26	234	193	24	260
12	248	10	266	125	22	250	135	23	261

Table 1: Expected computation time (in seconds) for different solar park capacity and ESS capacity.

1. the Expected Value strategy which solves a deterministic version of Problem 4 replacing random variables by their expected value;
2. the 2−stage strategy, detailed in section 4.3, which takes the decision v^{DA} minimizing the expected cost over $S_{MC} = 10$ scenarios $(\xi^s_{[T]})_{s\in[S_{MC}]}$. As S_{MC} is small, compared to the noise space, for computational reasons, we consider the median scenario with probability $\frac{1}{2}$;
3. the SDDP strategy in section 3.2 solves the continuous relaxation of Problem 4, and yields a solution taking into consideration the uncertainties on the whole horizon, but relaxing integrity.

	OPT			AR (in %)		
C^{PV} (MWc)	EV	2stage	SDDP	EV	2stage	SDDP
2	6067	**6023**	6038	1.6	**0.9**	1.1
4	5471	5483	**5451**	2.1	2.3	**1.7**
8	4552	4553	**4481**	4.2	4.2	**2.5**
12	3714	3691	**3641**	8.7	7.9	**6.7**

Table 2: Expected Cost (Opt) and Anticipative Regret (AR) of the solution obtained when finding v^{DA} with the different methods (EV, 2−stage, SDDP); parametrizing the operational problem with this v^{DA}; then solving the parametrized operational problem with MPC.

	EV			2stage			SDDP		
C^{PV} (MWc)	$I(v^{\mathrm{DA}}_{EV})$	$V(x_0; v^{\mathrm{DA}}_{EV})$	Opt	$I(v^{\mathrm{DA}}_{2S})$	$V(x_0; v^{\mathrm{DA}}_{2S})$	Opt	$I(v^{\mathrm{DA}}_{r})$	$V(x_0; v^{\mathrm{DA}}_{r})$	Opt
2	6002	65	6067	5830	193	**6023**	5659	379	6038
4	5369	102	5471	5123	360	5483	5102	349	**5451**
8	4357	195	4552	4073	480	4553	4043	438	**4481**
12	3394	320	3714	2965	726	3691	3094	548	**3642**

Table 3: We obtain $v^{\mathrm{DA}}_{EV}, v^{\mathrm{DA}}_{2S}, v^{\mathrm{DA}}_{r}$ by solving the problem respectively with the EV strategy, 2−stage programming and SDDP; then we parametrize and solve the operational problem with MPC for each v^{DA}.

From table 2, reporting simulated cost and anticipative regret of the various heuristics, we observe that, except for the instance with less uncertainties (first line), the day-ahead energy purchases determined with SDDP yield a lower expected cost as well as a lower anticipative regret than those determined with 2−stage programming or the EV strategy. As uncertainties grow (from top to bottom on the table), the anticipative regret increases and the gap between the AR of EV and the one of SDDP gets wider. Indeed, in the instance with a solar park of 4MWc, the anticipative regret is 0.4% lower for SDDP whereas it is 2% lower for the instance with more uncertainties (solar park of 12MWc).

On table 3 we separate design costs $I(v^{\mathrm{DA}})$ from operational costs $V(x_0; v^{\mathrm{DA}})$ for all instances solved. Whereas the EV strategy essentially pays energy in advance, the two-stage and SDDP strategies have lower design costs and buy more energy in real-time. This can be explained because a stochastic approach is looking for a trade-off between initial and recourse decisions. Assume that we have more energy than predicted, this extra energy comes for free and we better not have bought too much energy in advance, forcing us to throw this extra energy away (we can't charge the battery more than what is allowed). On the contrary, if we have less energy than predicted, we must either adapt the production plan (which might be possible) or buy energy in real-time which is not that much more expensive than if we bought it in advance (110% of day-ahead prices). Thus, we understand that in this problem, it is more efficient to underestimate the quantity of energy to buy from the main grid, as we have more to gain if the solar realization exceeds its prediction than we have to lose in the opposite case.

We give some additional insights on the various strategies in figs. 5 to 8. In fig. 5, we illustrate the day-ahead purchases over time. As expected, the day-ahead purchases are concentrated in the night and early morning, when energy from the grid is cheaper and no solar energy is available. Note that contrary to other approaches where day-ahead purchases are first-stage decisions, the anticipative day-ahead purchases are scenario dependent. Thus, we plot their expectation, which leads to a smoother function. Indeed, the minimum production constraint induces a discontinuity in the energy-load, and thus the day-ahead purchases. This is also reflected in fig. 6, where we plot the expected number of machines (out of 3). This is caused by the day-ahead purchases which shape the production: if energy has been bought for 2 machines, turning a third one on would be costly unless the available solar energy can cover it.

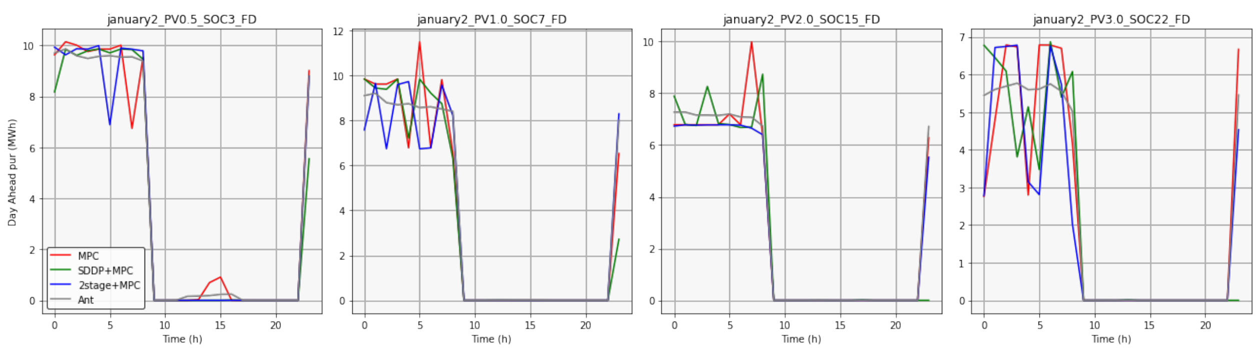


Figure 5: Day-ahead purchases y (in MWh) over time with the different methods (EV, 2−stage, SDDP). Averaged anticipative's day ahead purchased are also given.

Moreover, on fig. 7, we plot the expected battery storage (thick lines) over time for each method and the standard deviation in dashed lines. Notably, the anticipative solution doesn't use the battery as much as the other methods: indeed, as it is aware of the exact amount of solar energy that will be available, it can adapt the production of early stages precisely and does not need the flexibility to compensate for uncertainties. We can observe that the EV strategy always makes more use of the battery than the stochastic strategies (2−stage and SDDP). This confirms that a deterministic approach needs more flexibility to recover a good solution than a stochastic one.

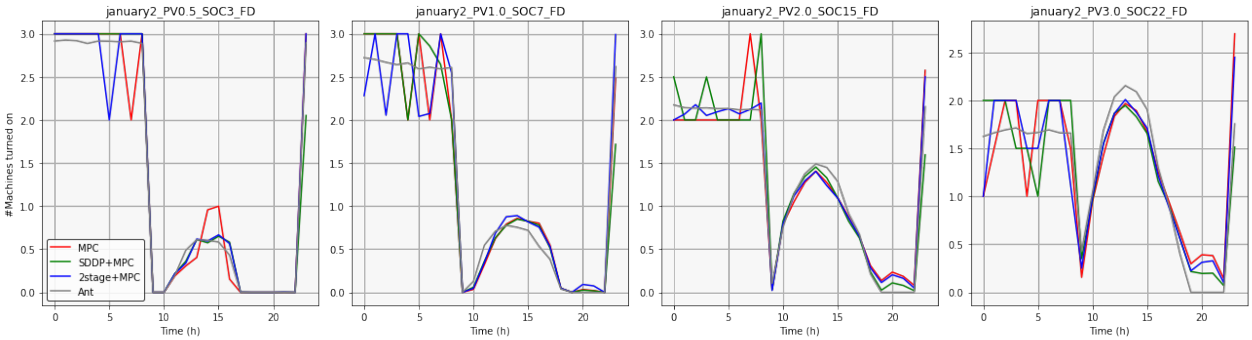


Figure 6: Expected number of machines turned on (out of 3) over time with the different methods (EV, 2−stage, SDDP) and with the anticipative solution.

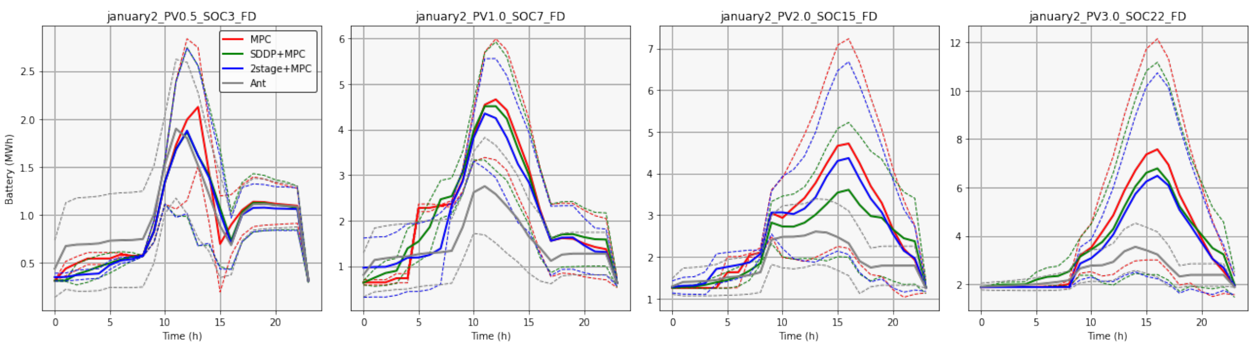


Figure 7: Expected trajectory of the battery (in MWh) over time with the different methods (EV, 2−stage, SDDP) and with the anticipative solution.

Finally, we can see in fig. 8 the cumulated stocks produced over time for each method. We observe that as uncertainties grow (from left to right), the stochastic solution (green line) gets closer to the anticipative production (grey line).

5.4 Results on stagewise dependent scenarios

In our use case, the only uncertainty lies in the day-ahead forecast error of solar production. This forecast error was based on an advanced statistical model that we consider as ground truth. As the aim of this work is to compare various methodological approaches we considered a simpler model with stagewise independent residual errors (see section 3).

As this assumption is not strictly satisfied by the advanced statistical model, we simulated the strategies on scenarios obtained from the advanced statistical model (see fig. 9). We find that, for these stagewise dependent scenarios, MPC is still the best method for the intraday problem, and the SDDP approach gives significantly better results than the EV strategy for the day-ahead problem (see fig. 9). We also observe that the expected optimal value of the SDDP approach is close to the expected anticipative optimal value. Thus, even if it would be possible to use a more advanced statistical model to train the SDDP approach (*e.g.,* autoregressive

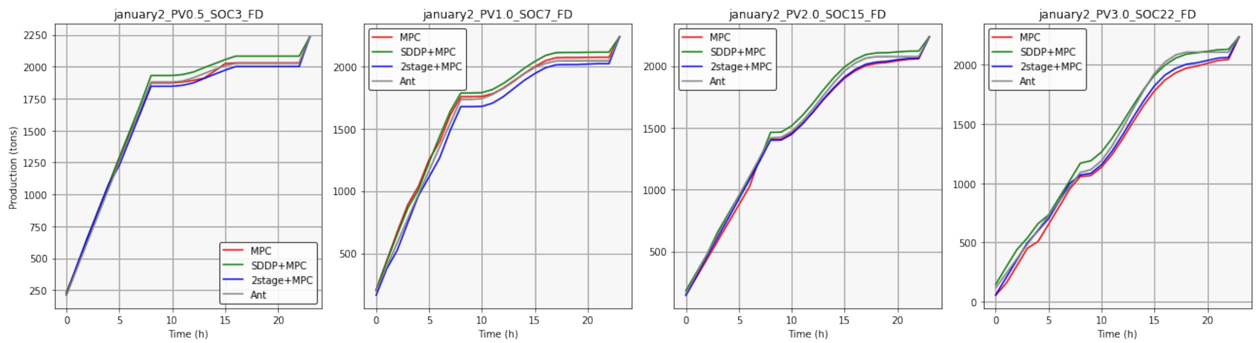


Figure 8: Evolution of the cumulated product stocks over time with the different methods (EV, 2−stage, SDDP) and with the anticipative solution.

or Markov Chain models), we do not believe that it would lead to significant improvements in our use case.

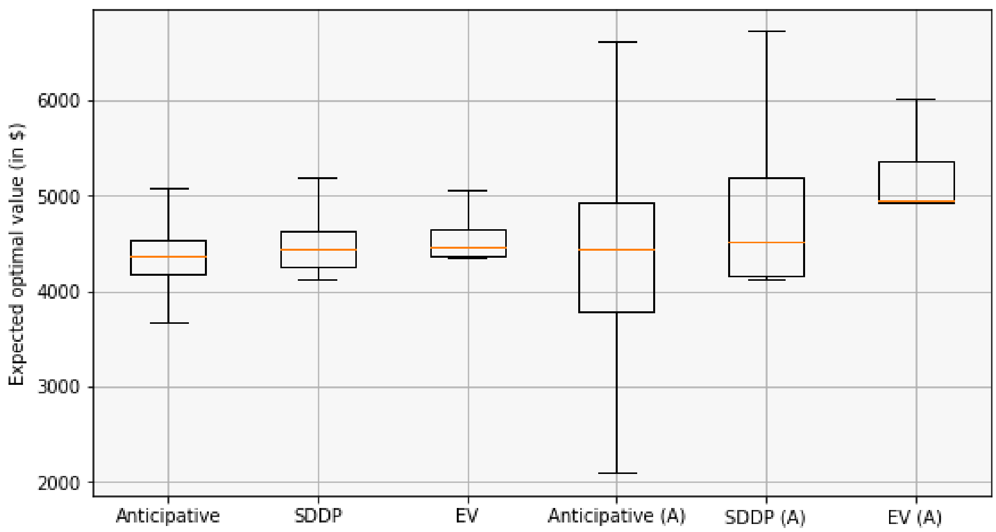


Figure 9: Expected Cost (Opt) of the solutions obtained when solving the operational problem parametrized by day-ahead purchases v^{DA} of the Anticipative, EV and SDDP strategies, first on a thousand scenarios with independence assumption; then on a thousand scenarios from the advanced statistical model (A).

6 Conclusion

In this paper, we considered a common problem in the industry: jointly optimizing the production planning and the energy supply of a factory, considering intra-day and day-ahead decisions. For ecological reasons we included renewable energies, leading to a stochastic optimization problem. We proposed various solution methodologies and compared them on a realistic industrial case study.

The main difficulty of the production plan comes from binary variables that cannot be relaxed. They are crucial to modeling the hard constraints of the problem. Integrating energy operations into the problem obliges us to handle uncertainties. In all the proposed algorithms to solve the problem, a trade-off must be found between relaxing integrity and relaxing information. For instance, MPC is fully deterministic whereas SDDP solves the continuous relaxation of the stochastic problem. In the tests we have conducted, we found that the right balance depends on the problem.

Indeed, for the intraday problem, where strategic decisions are given, we saw that using the Model Predictive Control algorithm, which consists in replacing the stochastic variables with deterministic ones, and reevaluating decisions at each stage, get the best results. However, for the day-ahead problem, we saw that solving the relaxed version of the problem with SDDP yields better strategic decisions as it is more aggressive in its day-ahead buying decisions. Indeed, having too much energy is less costly than having too little. Other methods were considered but did not yield interesting results in our case. In particular, two-stage approaches were not providing better solutions than deterministic MPC; and discretized dynamic programming was too slow. Finally, the look-ahead approach computes a feasible and reasonable solution from SDDP cuts. Although it does not beat MPC in this specific setting, the method shows promise as a viable compromise between relaxing integrity and achieving fast computational time.

To conclude, remember that we did not discuss the investment problem. Indeed, the return on investment is difficult to compute, as it highly depends on decarbonization subsidies or tax incentives as well as on the evolution of the energy markets. Recent events in Ukraine have shown that energy prices are volatile and unpredictable, especially in the long term. However, with conservative estimates, we obtain a return on investment of around 10% for solar parks. In our setting, investment in storage is not profitable. Nevertheless, if we allow buying and selling energy at the given prices, which is not completely realistic, the battery would be quite profitable. Without reselling energy, the profitability of storage also depends on the demand load: a high load requiring the machines to be on during peak prices would make the battery profitable. These investment aspects would require further investigation.

Statements and Declarations

The authors declare they have no financial interests.

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