DECOMPOSITION-COORDINATION METHOD FOR THE MANAGEMENT OF A CHAIN OF DAMS

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ABSTRACT. We study the management of a chain of dam hydroelectric production where we consider the expected gain stemming from the production as the criterion to maximize. However solving directly the problem by Dynamic Programming approach can be numerically impossible because of the so-called curse of dimension.Consequently we will use some decomposition-coordination method on this problem. However if decomposition-coordination methods are well known in a deterministic setting, few results are available in a stochastic setting.

We will present a simple problem with three dams, that can be solved by dynamic programming, and the Dual Approximate Dynamic Programming (DADP) decomposition method we are using on this problem. As we have the exact solution of the problem, we can present a torough study of the numerical properties of DADP.

Hydroelectricity is the main renewable energy in many countries. It provides a clean (no greenhouse gases emissions) and fast-usable energy that is cheap and substitutable for the thermal one. It is all the more important to ensure its proper use that it comes from a shared limited ressource: the reservoir water. This is the dam hydroelectric production management purpose.

Most dams are interconnected in a hydroelectric valley, that is the water turbined by one dam is going as an inflow in another one. Thus the dimension of the problem is multiplied by the number of dams in the valley. As it is well known, the curse of dimensionality forbids to use Dynamic Programming for hydraulic valley of more than 5 dams [1]. Consequently we have to use an approximate algorithm, like a price decomposition method. However prices in a stochastic setting would be stochastic processes, and thus would be intractable. We approach the problem by replacing the prices by their conditional expectation with respect to some information variable, and show the numerical properties of this method.

1. INTRODUCTION

We are interested in presenting a method of decomposition-coordination in a stochastic setting applied to the management of a chain of dams. There exists some methods to address this problem of hydroelectric valley management like Progressive-Hedging (see [6]), Aggregation (see [8]) or Stochastic Dual Dynamic Programming (SDDP, see [5] for the original presentation, and [7] for a recent analysis of the method. Most of theses methods are scenario-tree based, which imply some serious limitations when you want to obtain a policy. We are going to work on this problem by using some decomposition-coordination methods, as presented in [3]. However the extension to a stochastic setting is not really simple (see [2] for example). Recently an algorithm has been proposed in [1] that came from the price-decomposition method, and we are applying it on the optimal management of a chain of dam problem.

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[1]Indeed with our choice of discretization we need about 90 seconds to solve a 3−dam valley, thus a 5 dam valley would need about 4 days of computing.

1.1. **Description of the problem.**
We consider a chain of N dams where the outflows of the dam i are inflows for the dam $i+1$. We consider that all dams are controlled by the same firm, and thus we want to optimize the sum of the payoffs.

We present here the problem we are addressing, as shown in figure 1.

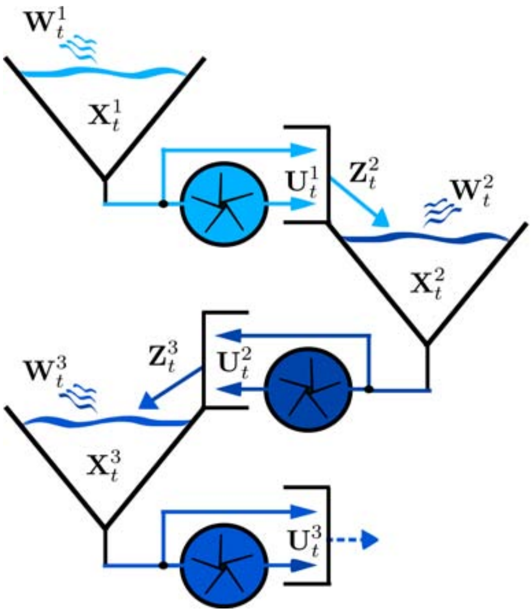

FIGURE 1. The river chain model

1.1.1. *Dynamics of the dam.*

Let time t vary in $\{0, ..., T\}$. For all dams $i \in \{1, ..., N\}$, the following positive real valued random variables are defined on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$:

- x_t^i, the *storage level* of dam i at the beginning of period $[t, t+1[$, (*state*)
- u_t^i the *hydroturbine* outflows of dam i during $[t, t+1[$, (*control*)
- w_t^i and p_t^i, the *external inflows* and the *production earnings* of dam i during $[t, t+1[$. (*noise*)

We assume that the noise are random variables that are mutually and step by step independent. They are uniformly distributed on a discrete set. The dynamics of the reservoir storage level reads, for the first dam of the chain :

$$\begin{aligned} x_{t+1}^1 &= f_t^1(x_t^1, u_t^1, w_t^1, 0) , \\ &= x_t^1 - u_t^1 + w_t^1 . \end{aligned}$$

And for any other dam $i > 1$ we have

$$\begin{aligned} x_{t+1}^i &= f_t^i(x_t^i, u_t^i, w_t^i, z_t^i) , \\ &= x_t^i - u_t^i + w_t^i + z_t^i , \end{aligned}$$

where

$$z_t^i = x_t^{i-1} - u_t^{i-1} + w_t^{i-1} + z_t^{i-1} \qquad (1)$$

is the water inflows in dam i coming from dam $i-1$, it is also the total outflows of dam $i-1$.

The bound constraints are:e

$$\underline{x}_{t+1} \leq x_{t+1} \leq \overline{x}_{t+1} \quad \text{and} \quad \underline{u}_t \leq u_t \leq \overline{u}_t, \quad \forall t \in \{0, \dots, T-1\}. \tag{2}$$

Moreover we assume the *Hazard-Decision* information structure (u_t^i is chosen once w_t^i is observed), so that $\underline{u}^i \leq u_t^i \leq \min\left\{\overline{u}^i, x_t^i + w_t^i + z_t^i - \underline{x}^i\right\}$.

1.1.2. *Objective function.*

We are considering the multiple step management of a chain of dams, each dam produces electricity, with an efficiency coefficient η^i, that is sold at the same price. Thus the hydroelectric valley obeys the following valorization[2] mechanism

$$\sum_{i=1}^{N} \sum_{t=0}^{T-1} -p_t \eta^i u_t^i + \varepsilon(u_t^i)^2 + K_i(x_T) , \tag{3}$$

where K_i is a function valorizing the remaining water at time T in the dam i. The $\varepsilon(u_t^i)^2$ term is here to represent some non-linearity in the efficiency of turbines as well as numerically stabilize the problem by making it strongly convex. As this criterion is random, we choose to minimize the expected cost

$$\mathbb{E}\left[\sum_{i=1}^{N} \sum_{t=0}^{T-1} \underbrace{-p_t \eta^i u_t^i + \varepsilon(u_t^i)^2}_{=L_t^i(x_t^i, u_t^i, w_t^i, z_t^i)} + K_i(x_T)\right] . \tag{4}$$

Let $\mathbb{F} = \left\{\mathcal{F}_t\right\}_{t=0,\dots,T}$ be the filtration of past noises:

$$\mathcal{F}_t = \sigma\big(W_0, \dots, W_t\big) , \quad \text{with } W_t = \left\{W_t^1, \dots, W_t^N\right\} .$$

Thus the stochastic optimization problem we are solving reads

$$\min_{(X,U,Z)} \mathbb{E}\left(\sum_{i=1}^{N}\left(\sum_{t=0}^{T-1} L_t^i\big(X_t^i, U_t^i, W_t^i, Z_t^i\big) + K^i\big(x_T^i\big)\right)\right), \tag{5a}$$

subject to:

$$X_{t+1}^i = f_t^i(X_t^i, U_t^i, W_t^i, Z_t^i) , \quad \forall i , \quad \forall t , \tag{5b}$$

$$Z_t^{i+1} = g_t^i(X_t^i, U_t^i, W_t^i, Z_t^i) , \quad \forall i , \quad \forall t , \tag{5c}$$

as well as measurability constraints:

$$U_t^i \preceq \mathcal{F}_t , \quad \forall i , \quad \forall t . \tag{5d}$$

1.1.3. *Some remarks about the optimization problem.*

The noises W_t^i are independent over time, so that the problem can be theoretically solved by Dynamic Programming (**DP**). The resulting optimal feedback laws at time t depend on the current states of the dams:

$$U_t^{i\sharp} = \gamma_t^i\big(X_t^1, \dots, X_t^i, \dots, X_t^N\big) , \quad \forall i , \quad \forall t .$$

[2] As usual in optimization we choose to minimize the opposite of the gain.

However **DP** is subject to the curse of dimensionality: the method is not numerically tractable as soon as $N \geq 5$. And thus we have to find another numerical solution. Let's note that the coupling between the dams arises only from Equation (1) : $Z_t^{i+1} = g_t^i(X_t^i, U_t^i, W_t^i, Z_t^i)$. And this is the constraint we will dualize in order to use price-decomposition method on it.

2. Price decomposition and Uzawa's algorithm

The main idea of the price decomposition of problem (5c) is to see Z_t^i as an independant variable as shown in figures 2 (a) and 2 (b).

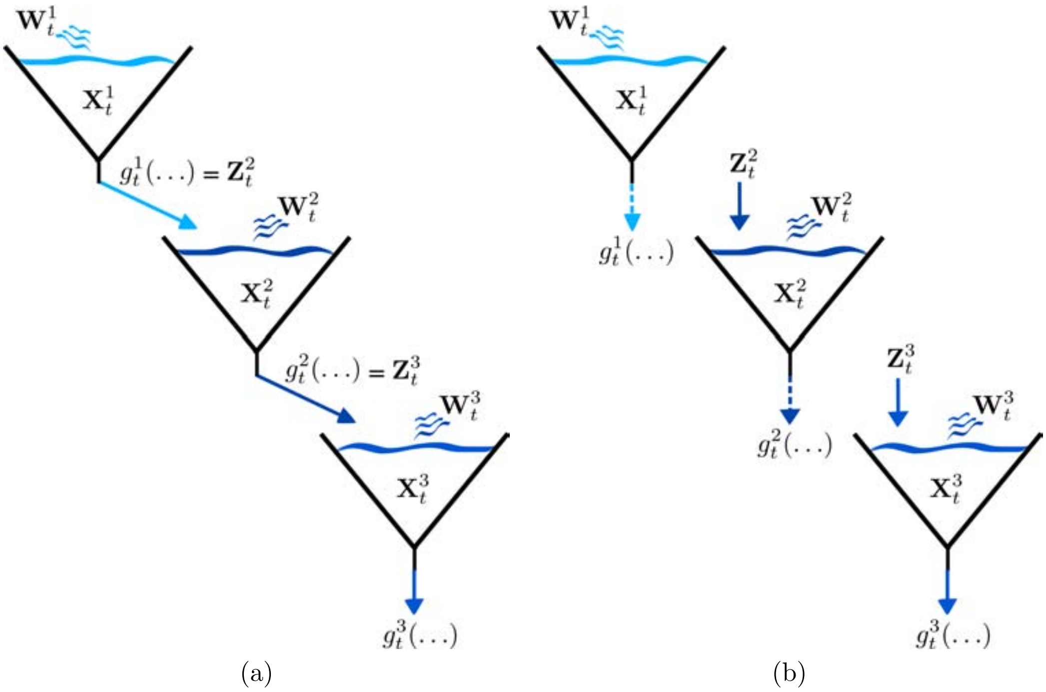

Figure 2. (a): Whole Problem (b): Decomposed Problem

2.1. Dualization of the coupling constraint.

We aim at dualizing Constraint (1) and at solving the Problem (5) by using the Uzawa algorithm: at iteration k, the associated multiplier is a fixed $\mathcal{F}_t$-measurable random variable $\left(\lambda_t^{i+1}\right)^{(k)}$, and the term (under the expectation) induced by duality in the cost function is

$$\left(\lambda_t^{i+1}\right)^{(k)} . \left(Z_t^{i+1} - g_t^i\left(X_t^i, U_t^i, W_t^i, Z_t^i\right)\right) ,$$

(note that $\left(\lambda_t^{i+1}\right)^{(k)}$ is related to X_t^i). It can be decomposed as

- $\left(\lambda_t^{i+1}\right)^{(k)} . Z_t^{i+1}$: term pertaining to dam $i+1$.
- $-\left(\lambda_t^{i+1}\right)^{(k)} . g_t^i\left(X_t^i, U_t^i, W_t^i, Z_t^i\right)$: term pertaining to dam i.

Finally, the following term is added to the cost of dam i

$$\left(\lambda_t^{i}\right)^{(k)} . Z_t^i \; - \; \left(\lambda_t^{i+1}\right)^{(k)} . g_t^i\left(X_t^i, U_t^i, W_t^i, Z_t^i\right) .$$

Consequently the algorithm is done as follow :

(1) we fix multipliers $\left(\lambda_t^{i}\right)^{(k)}$ for all i and t,

(2) we have to solve N problems with only one dam,
(3) we update the multiplier by a gradient step.

2.2. **Optimization subproblem at iteration k.**

Consequently optimization problem associated to dam i at iteration k of the Uzawa algorithm is:

$$\min_{(X^i,U^i,Z^i)} \mathbb{E}\left(\sum_{t=0}^{T-1}\left(L_t^i(X_t^i,U_t^i,W_t^i,Z_t^i) + (\lambda_t^i)^{(k)}.Z_t^i - (\lambda_t^{i+1})^{(k)}.g_t^i(X_t^i,U_t^i,W_t^i,Z_t^i)\right) + K^i(x_T^i)\right), \quad (6a)$$

subject to:

$$X_{t+1}^i = f_t^i(X_t^i,U_t^i,W_t^i,Z_t^i)\ , \ \forall t\ , \quad (6b)$$

and the measurability constraints:

$$U_t^i \preceq \mathcal{F}_t \quad \text{and} \quad Z_t^i \preceq \mathcal{F}_t\ , \ \forall t\ . \quad (6c)$$

With boundary conditions: $Z_t^1 \equiv 0$ and $\lambda_t^{N+1} \equiv 0$.

This problem is a one dimensional dam problem and can be solved by **DP** or by any other method.

3. Dual Approximate Dynamic Programming (**DADP**)

3.1. **DADP principle.**

The presence of the random variables $(\lambda_t^i)_{t=0,\dots,T-1}^{(k)}$ prevents us to use **DP** unless the property of independence (of the λ) over time is verified, which is not the case.

The idea of **DADP**, as presented in [1] and [4] is to replace the (known) multiplier $(\lambda_t^i)^{(k)}$ by its conditional expectation w.r.t. a chosen information variable Y_t^i, namely $\mathbb{E}\left((\lambda_t^i)^{(k)} \mid Y_t^i\right)$, or equivalently to replace (5c) by

$$\mathbb{E}\left(Z_t^i - g_t^{i-1}(X_t^{i-1},U_t^{i-1},W_t^{i-1},Z_t^{i-1}) \;\middle|\; Y_t^i\right)\ . \quad (7)$$

Let's note that this approximation is a relaxation of the problem (as the constraint is loosened), and thus the strategies that we derive may not be admissible, even if the algorithm converges. Thus we still have to construct an admissible strategy from the one we obtain with the DADP algorithm.

In practice, Y_t^i is a short-memory process that will enter the state variables of the subproblems. Possible choices for Y_t^i are:

(1) $Y_t^i \equiv \text{const}$: we deal with the constraint in expectation,
(2) $Y_t^i = W_t^{i-1}$: we incorporate the noise W_t^{i-1} in Subproblem i,
(3) $Y_t^i = \widetilde{f}_t^{i-1}(Y_{t-1}^i, W_t^{i-1})$: we mimic the dynamics of X_t^{i-1}.

We have choosen to explore the case where Y_t^i mimic X_t^{i-1}.

3.2. **Optimization subproblem in DADP.**

The conditional expectation $\mathbb{E}\left((\lambda_t^i)^{(k)} \mid Y_t^i\right)$ corresponds to a function $(\varphi_t^i)^{(k)}(Y_t^i)$ which can be pre-computed (by a least-sqare fitting on some known trajectories for example). Consider the choice: $Y_t^i = \widetilde{f}_t^{i-1}(Y_{t-1}^i, W_t^{i-1})$. Subproblem i writes:

$$\min \mathbb{E}\left(\sum_{t=0}^{T-1}\left(L_t^i(X_t^i,U_t^i,W_t^i,Z_t^i) + (\varphi_t^i)^{(k)}(Y_t^i).Z_t^i - (\varphi_t^{i+1})^{(k)}(Y_t^{i+1}).g_t^i(X_t^i,U_t^i,W_t^i,Z_t^i)\right) + K^i(x_T^i)\right), \tag{8a}$$

subject to (measurability constraints are omitted):

$$X_{t+1}^i = f_t^i(X_t^i,U_t^i,W_t^i,Z_t^i)\ , \tag{8b}$$

$$Y_t^i \ = \widetilde{f}_t^{i-1}(Y_{t-1}^i,W_t^{i-1})\ , \tag{8c}$$

$$Y_t^{i+1} = \widetilde{f}_t^i(Y_{t-1}^{i+1},W_t^i)\ . \tag{8d}$$

The state is a 3-dimensional vector, consequently Dynamic Programming can be used to solve the sub-problem.

3.3. **detailed algorithm.**

We give here a formal presentation of the algorithm. First the initialization of the algorithm should be done as follow

- We fix some random particles (that is some trajectories of the noise) $(W_t^l)_{t\in[0,T]}$ that will be used throughout the algorithm.
- We initialize $\left(\lambda_t^i\right)^{(0)}$ as deterministic well chosen constants (zero by default), and $\left(\varphi_t^i\right)^{(0)}$ as constant functions.
- We define $\left(Y_t^i\right)^{(0)} := 0$

A better starting point for λ_t^i could be found from the optimal solution on the mean scenario for example.

Then at the beginning of iteration k we should have defined

- A noise variable ξ_t^i.
- A variable of information $\left(Y_t^i\right)^{(k)}$ which should be an (uncontrolled process)

$$Y_t^i = \widetilde{f}_t^i(Y_{t-1}^i,\xi_t^i)\ .$$

- A function $\left(\varphi_t^i\right)^{(k)}$ such that

$$\left(\varphi_t^i\right)^{(k)}(y) \approx \mathbb{E}\left(\left(\lambda_t^i\right)^{(k)} \mid \left(Y_t^i\right)^{(k)} = y\right)$$

For each i we solve

$$\begin{aligned}
\min_{X^i,U^i,Z^i}\quad & \mathbb{E}\Bigg[\sum_{t=0}^{T} L_t^i(X_t^i,U_t^i,W_t^i) + (\varphi_t^i)^{(k)}(Y_t^i).Z_t^i \\
& \qquad - (\varphi_t^{i+1})^{(k)}(Y_t^{i+1}).g(X_t^i,U_t^i,W_t^i,Z_t^i)\Bigg] \\
& X_{t+1}^i = f_t^i(X_t^i,U_t^i,Z_t^i,W_t^i) \\
& Y_{t+1}^i = (\tilde{f}^i)_t^{(k)}(Y_t^i,\xi_t^i) \\
& Y_{t+1}^{i+1} = (\tilde{f}^{i+1})_t^{(k)}(Y_t^{i+1},\xi_t^{i+1}) \qquad\qquad U_t^i \preceq \mathcal{F}_t \\
& Z_t^i \preceq \mathcal{F}_t
\end{aligned}$$

This gives us some optimal feedback laws

- $(\gamma_t^i)^{(k)}\left(X_t^i, Y_t^i, Y_t^{i+1}, W_t^i, \xi_t^i, \xi_t^{i+1}\right) \quad \rightsquigarrow \quad U_t^i$
- $(\eta_t^i)^{(k)}\left(X_t^i, Y_t^i, Y_t^{i+1}, W_t^i, \xi_t^i, \xi_t^{i+1}\right) \quad \rightsquigarrow \quad Z_t^i$

that are used with $(X_t^{i,l})^{(k)}$, $(U_t^{i,l})^{(k)}$, $(Z_t^{i,l})^{(k)}$, $(Y_t^{i,l})^{(k)}$, $(Y_t^{i+1,l})^{(k)}$, to compute

$$(U_t^{i,l})^{(k)} = (\gamma_t^i)^{(k)}\left((X_t^i)^{(k)}, (Y_t^i)^{(k)}, (Y_t^{i+1})^{(k)}, W_t^{i,l}, \xi_t^{i,l}, \xi_t^{i+1,l}\right)$$
$$(Z_t^{i,l})^{(k)} = (\eta_t^i)^{(k)}\left((X_t^i)^{(k)}, (Y_t^i)^{(k)}, (Y_t^{i+1})^{(k)}, W_t^{i,l}, \xi_t^{i,l}, \xi_t^{i+1,l}\right)$$

and

$$(X_{t+1}^{i,l})^{(k)} = f_t^i\left((X_t^{i,l})^{(k)}, (U_t^{i,l})^{(k)}, (Z_t^{i,l})^{(k)}, W_t^{i,l}\right)$$
$$(Y_{t+1}^{i,l})^{(k)} = (\tilde{f}^i)_t^{(k)}\left((Y_t^{i,l})^{(k)}, \xi_t^{i,l}\right)$$
$$(Y_{t+1}^{i+1,l})^{(k)} = (\tilde{f}^{i+1})_t^{(k)}\left((Y_t^{i+1,l})^{(k)}, \xi_t^{i+1,l}\right)$$

And finally we can

- Update of the prices trajectories:
$$(\lambda_t^{i+1,l})^{(k+1)} := (\lambda_t^{i+1,l})^{(k)} + \rho^{(k)}(\Delta_t^{i,l})^{(k)} ,$$
with $(\Delta_t^{i,l})^{(k)} := (Z_t^{i+1,l})^{(k)} - g_t^i\big((X_t^{i,l})^{(k)}, (U_t^{i,l})^{(k)}, W_t^{i,l}, (Z_t^{i,l})^{(k)}\big)$.
- Define a new information dynamics $(\tilde{f}^i)_t^{(k+1)}$.
- Simulate $(Y_t^{i,l})^{(k+1)}$.
- Make a regression of $(\lambda_t^{i,l})^{(k+1)}$ on $(Y_t^{i,l})^{(k+1)}$ to obtain
$$(\varphi_t^i)^{(k+1)}(y) \approx \mathbb{E}\left((\lambda_t^i)^{(k+1)} \;\middle|\; (Y_t^i)^{(k+1)} = y\right) .$$

which terminate step k.

3.4. **Heuristic to construct an admissible solution.**

Once the algorithm has converged we have some feedbacks laws that must verify the constraint (7)

$$\mathbb{E}\left(Z_t^i - g_t^{i-1}(X_t^{i-1}, U_t^{i-1}, W_t^{i-1}, Z_t^{i-1}) \;\middle|\; Y_t^i\right) .$$

which means that the mechanical constraint $Z_t^i = g_t^{i-1}\big(X_t^{i-1}, U_t^{i-1}, W_t^{i-1}, Z_t^{i-1}\big)$ is not verified. Consequently one has to define an heuristic to turn this strategies into an admissible one. As we have choosen Y_t^i such that it should mimics X_t^{i-1} we can construct an approximate Bellman's value function for the global problem as the sum of the Bellman's value function of each subproblem where Y_t^i is replaced by Y_t^i. Consequently we obtain a global admissible strategy by doing a one time step optimization of the global problem.

More precisely if we write $V_t^i(X_t^i, Y_t^i, Y_t^{i+1})$ the Bellman's function obtained for subproblem i when the DADP algorithm has converged, we define $\tilde{V}_t(x_t) = \sum_{i=1}^N V_t^i(X_t^i, X_t^{i-1}, X_t^i)$, with the convention that $x_0^t = 0$. Then the control we choose at time t when the chain of dams is in the state x_t is the optimal solution of

$$\min_{u_t} \sum_{i=1}^{N} L_t^i(X_t^i, U_t^i, W_t^i, Z_t^i) + \tilde{V}(f(x_t, u_t, w_t)) . \tag{9}$$

Let us note that this problem is global on the chain, but only done on one time-step, and thus numerically tractable.

4. Numerical Results

In order to make some interesting study of this method we have choosen a problem with three dams (i.e :$N = 3$), in order to be able to solve explicitely the problem by dynamic programming. Thus we can compare the solution of DADP algorithm with the exact solution. Moreover we have done some statistical studies on the optimal solution in order to choose wisely.

4.1. **Numerical parameters of the problem.** The characteristics of the study are:

- $\{\min, \max\}$ bounds on $\mathbf{X}_t^i = \{0, 80\}$ hm^3, $\forall(i,t)$;
- time steps number $T = 12$ (one step a month over a year);
- $\{\min, \max\}$ bounds on $\mathbf{U}_t^i = \{0, 40\}$ $hm^3 month^{-1}$, $\forall(i,t)$.

The stochastic universe is finite. The noise processes are white and uniformly distributed and the inflows at the three dams reservoirs are correlated. The simulation is based on 500 inflows scenarios. Figure 3 represents six inflows scenarios at dam 1, dam 2 and dam 3 and Figure 4 represents the price scenario. We set $\eta^1 = \eta^2 = \eta^3$.

4.2. **Optimal solution.** We solve the problem by using the dynamic programming algorithm.

- expected total gain $= 1.470 \times 10^6$ €;

The Figure 5, part a, shows six representative storage level trajectories (in hm^3, over 12 months) that we obtained by the integration of the dynamic programming-computed strategy.

4.3. **DADP solution.** We solve the problem by using the DADP algorithm. The multipliers processes are estimated by their expected values. The scenarios which are used to run the Uzawa algorithm are different from those which are used to simulate the computed strategy. The expected values of the multipliers converge (Figure 6) and the results are:

- expected total gain $= 1.405 \times 10^6$ €;
- iterations number = about 3000.

The approximation that we make by estimating the multipliers as their expected values leads to a loss of about 1%. This is all the more promising that we use the simplest information variable.

The simulated storage level trajectories appear quite similar in Figure 5 to the optimal ones. Of course, they are not exactly the same and we can see some significant differences but their global aspects correspond. It is then interesting to notice, thanks to Figure 5, the fact that the storage levels at dam 2 and dam 3 are likely to be higher with the optimal strategy than with the approximated one; whereas it does not at dam 1. This is due to the misestimation of the coupling between the reservoirs which is relative to the combination of the approximation of the multipliers by their expected values and the use of a heuristic to make the strategy admissible.

The spatial correlation between the inflows noise random variables and the sharing of a common price between the dams may explain the fact that the suboptimal gain stay pretty close to the optimal one, however. By the way, we observe in Figure 7 that the strategies aren't far from being almost surely the same for dam 1 and for dam 2 as the prices are significantly interesting or not. We see indeed, that the DP-computed and the DADP-computed strategies quite correspond at time 3, as the price is the highest, and at time 8, as it is the lowest. Moreover, we see that the strategies are almost surely (with respect to the 500 scenarios) the same for dam 3 from time 1

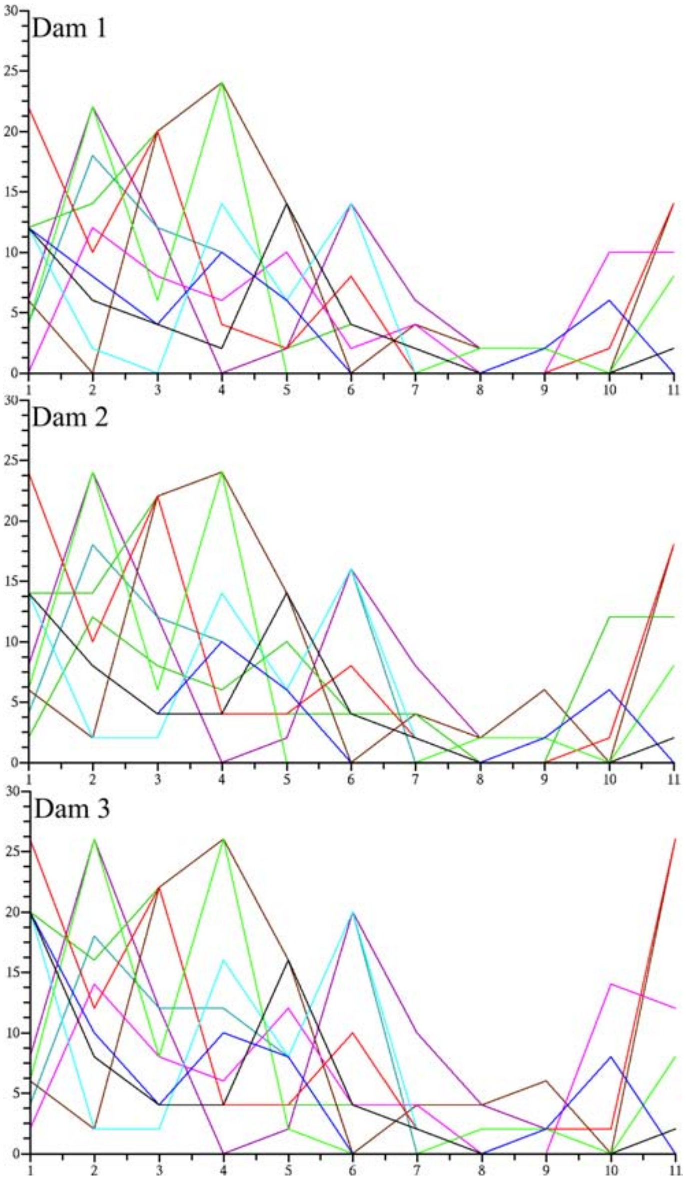

FIGURE 3. Six inflows scenarios

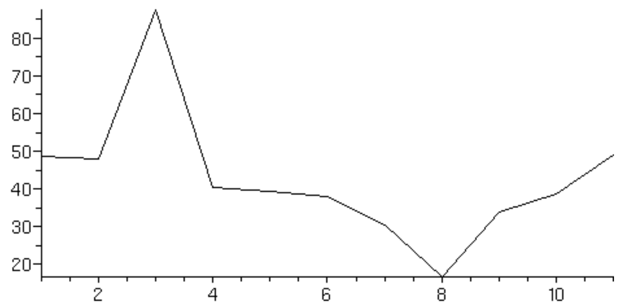

FIGURE 4. Deterministic price trajectory

to time 6. This is explained by the abundance of water at dam 3 during $\{1, \ldots, 6\}$ which leads to the optimal strategy $U_\tau^3 = \overline{u}_\tau$, $\forall \tau \in \{1, \ldots, 6\}$.

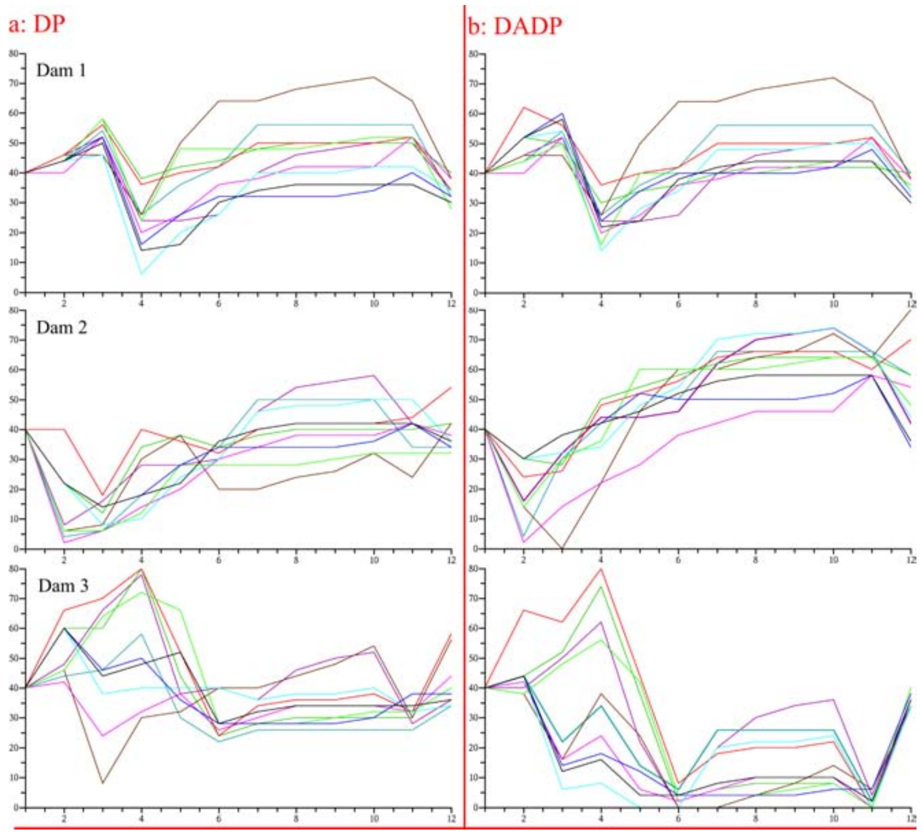

FIGURE 5. Six storage level trajectories

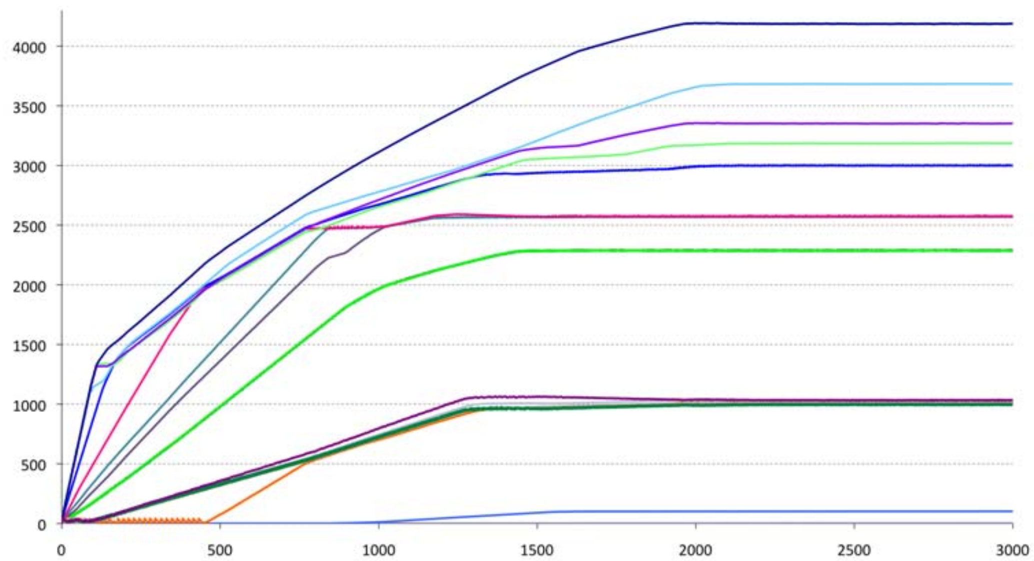

FIGURE 6. Convergence of the multipliers expected values (along 3000 iterations)

5. CONCLUSION

The frontal approach by dynamic programming to a problem like the optimization of an hydroelectric valley is not numerically tractable because of the so-called curse of dimensionality.

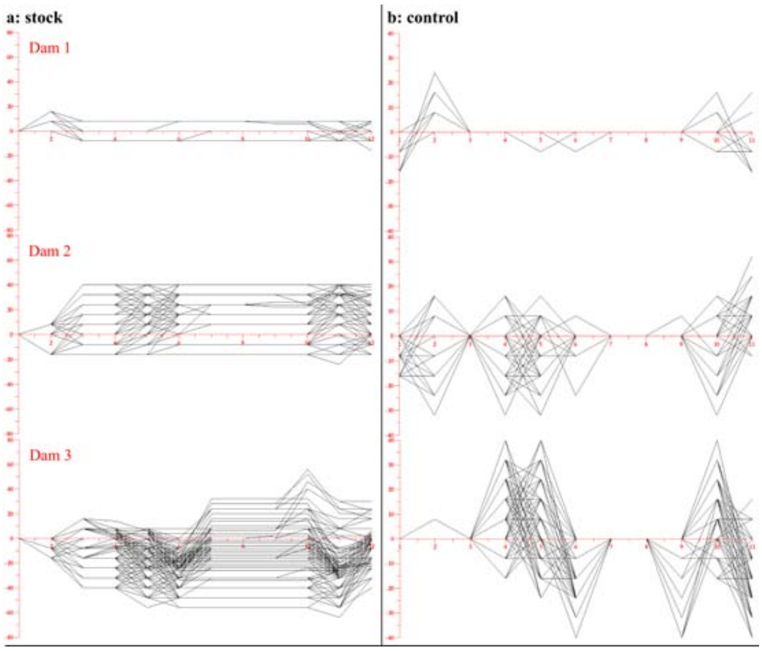

FIGURE 7. Differences in stock and controls on the 500 simulation scenarios ω

Decomposition-coordination approach can not be directly applied altogether as the probabilistic structure implies that each sub-problem would be as complicated as the original one. Thus DADP appears as a way of doing a price-decomposition approach by replacing the multiplier λ_t^i by its conditional expectation. Once the solution of the related problem is found we use an heuristic to obtain an admissible strategy from the one given by DADP algorithm.

Numerical results are quite encouraging, and statistical studies on the optimum λ_t^i give some good insights in order to choose a good information variable Y_t^i.

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